On the anomalous behavior of the Lagrangian structure function similarity constant inside dense canopies

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Abstract

The choice of the Kolmogorov constant ($C_0$) in Lagrangian Stochastic Models (LSMs) for canopy flows remains a subject of debate and uncertainty. This uncertainty stems from the fact that canopy flows are highly dissipative, lack a well-defined inertial subrange (ISR) in their energy cascade, and in the deeper layers of the canopy, the attenuation of turbulence can amplify finite Reynolds number effects on $C_0$. From the analysis here, it was shown that $C_0$ inside dense canopies is reduced relative to its value in the atmospheric surface layer (ASL) primarily due to wake production (a factor of 5), followed by finite Reynolds number effects (a factor of 1.5 at most). The short-circuiting of the energy cascade tends to increase $C_0$ though not enough to compensate for the other two reductions. These results are qualitatively consistent with theoretical predictions of a reduced $C_0$ with an increased anisotropy and localized acceleration when referenced to a homogeneous isotropic stationary turbulence. Simplified scaling arguments were proposed for each of these three effects and tested using flume experiments. The fact that $C_0$ may vary nonlinearly inside canopies complicates inverse estimates of $C_0$ that use fitting Lagrangian dispersion models (LDMs) to mean concentration measurements. The $C_0$ values inferred from such an approach were shown to be sensitive to the source location (especially inside the canopy) and concentration sampling points. On a positive note, the fact that $C_0$ may vary within the canopy does not require any revisions to the well-mixed condition because LDM are not sensitive to gradients in $C_0$. A phenomenological model that accounts for the vertical variation in $C_0$ as a function of the most elementary flow variables, the mean velocity and canopy adjustment length scale, is proposed but its general applicability remains to be tested.

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1. Introduction

Three-dimensional Lagrangian dispersion models (LDMs) of scalar transport within the canopy sublayer (CSL) of dense canopies are now being employed in numerous studies such as linking seed and pollen dispersal to wind statistics (Katul et al.,

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trace gas fluxes to their biological or chemical parent source or footprint calculations (Baldocchi, 1997; Baldocchi et al., 1999; Hsieh et al., 1997, 2000, 2003; Katul et al., 1997b; Kljun et al., 2002, 2003; Lai et al., 2002a, b; Markkanen et al., 2003; Rannik et al., 2003; Siqueira et al., 2002; Vesala et al., 2004), urban concentration predictions (de Haan et al., 2001; Rotach, 2001; Rotach et al., 2004; Roth, 2000), concentration fluctuations prediction (Cassiani et al., 2005c, 2007a, b) and more recently understanding chemo-tactic (and ‘infotaxis’) algorithms (Vergassola et al., 2007) along with odor dispersion (Schiffman et al., 2005), to name a few.

Among the defining syndromes of flows in the CSL are the vertical in-homogeneity and non-Gaussianity, the highly dissipative rate of turbulent kinetic energy and its non-monotonic vertical variation, the lack of a well-defined inertial subrange (ISR) in the energy cascade, and in the deeper layers of the canopy, the significant attenuation of turbulence (Cassiani et al., 2005c, 2007a; Finnigan, 2000; Poggi et al., 2006; Raupach, 1981, 1987, 1988, 1989a, b; Raupach and Thom, 1981) resulting in finite Reynolds number effects that may not be ignored in LDM calculations. These syndromes pose unique challenges to the standard application of LDM approaches that satisfy the so-called well-mixed condition inside canopies. While classical LDM permits accounting for vertical in-homogeneity in the flow statistics without violating the well-mixed condition (Thomson, 1987), the non-Gaussian characteristics and the fact that the energy cascade does not possess a clear ISR behavior remain problematic.

Numerous studies (Flesch and Wilson, 1992; Poggi et al., 2006) have already discussed the importance of non-Gaussian statistics on the LDM mean scalar concentration predictive skills with some noting minor improvements while others noting greater divergence in the data-model inter-comparison. However, few studies attempted to evaluate the lack of ISR scaling inside canopies on LDM calculations or finite Reynolds number effects, the subject of this study.

As a starting point, the lack of ISR scaling is assessed by considering how wake production and energy short-circuiting of the cascade modify the Eulerian energy spectrum within the canopy, and explore whether these modifications can be ‘absorbed’ by $C_0$, the similarity constant for the Lagrangian structure function. This is a convenient starting point because much of the recent results on finite Reynolds number effects using Direct Numerical Simulations (DNS) have been ‘encoded’ in relative variations in $C_0$ with the Taylor micro-scale Reynolds number ($R_L$) as shown in Table 1. Those DNS results can be used to explore the order-of-magnitude effects of finite Reynolds number on $C_0$ inside dense canopies and compare them with other effects. Furthermore, beyond the convenience of

<table>
<thead>
<tr>
<th>Source or reference</th>
<th>$C_0$</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>Similarity arguments in the equilibrium or log-layer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rodean (1991)</td>
<td>5.7</td>
<td>Diffusivity matching</td>
</tr>
<tr>
<td>Li and Taylor (2005)</td>
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<td>DNS extrapolated to high Reynolds numbers ($R_L$)</td>
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<td>Sawford (1991)</td>
<td>7.0</td>
<td>38–93</td>
</tr>
<tr>
<td>Pope (1994)</td>
<td>6.2</td>
<td>38–93</td>
</tr>
<tr>
<td>Sawford and Yeung (2001)</td>
<td>6.0</td>
<td>38–240</td>
</tr>
<tr>
<td>Yeung (2002)</td>
<td>6.4</td>
<td>38–234</td>
</tr>
<tr>
<td>Determined by fitting LSM to concentration data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pope (1994)</td>
<td>2.1</td>
<td>Lab (diffusion behind a line source)</td>
</tr>
<tr>
<td>Pope (1994)</td>
<td>3.5</td>
<td>Lab</td>
</tr>
<tr>
<td>Du et al. (1995)</td>
<td>2.5–3.5</td>
<td>Lab and ASL</td>
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<tr>
<td>Pope and Chen (1990)</td>
<td>5.5</td>
<td>Lab and ASL</td>
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<tr>
<td>Reynolds (1998)</td>
<td>5.5</td>
<td>Lab and ASL</td>
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<tr>
<td>Poggi et al. (2006)</td>
<td>5.5</td>
<td>Flume experiments, canopy flows</td>
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<tr>
<td>Cassiani et al. (2005a, b)</td>
<td>2.0</td>
<td>Water tank convective boundary layer</td>
</tr>
<tr>
<td>Cassiani et al. (2005a)</td>
<td>5.0</td>
<td>Wind tunnel, neutral ABL</td>
</tr>
<tr>
<td>Rotach et al. (2004)</td>
<td>1.0–3.0</td>
<td>Urban canopy (roof-top)</td>
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<td>Determined from LES runs for the atmospheric boundary layer</td>
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<tr>
<td>Anfossi et al. (2006)</td>
<td>2.5, 4.3, 4.5, 4.1, 5.1, 6.1</td>
<td>Strongly convective case for $u$, $v$, $w$</td>
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<td></td>
<td>4.5, 5.5, 6.1</td>
<td>Buoyant case for $u$, $v$, $w$</td>
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<td></td>
<td>2.9–3.5</td>
<td>Near-neutral case for $u$, $v$, $w$</td>
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<td>Trajectory experiments</td>
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<td>Hanna (1981)</td>
<td>2.2–6.1</td>
<td>ASL</td>
</tr>
<tr>
<td>Lien and D’Asaro (2002)</td>
<td>3.1–6.2</td>
<td>Oceanic boundary layer</td>
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<tr>
<td>Mordant et al. (2001)</td>
<td>2.9–3.5</td>
<td>Laboratory</td>
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<tr>
<td>Ouellette et al. (2006b)</td>
<td>6.3, 6.3, 4.5</td>
<td>Lagrangian tracer particles in a rotating tank (for $u$, $v$, $w$)</td>
</tr>
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</table>
lumping all such complex issues in $C_0$, there are practical reasons why adjusting $C_0$ should be pursued first. A large number of studies have already reported good agreement between mean scalar concentration profile predictions using the standard LDM and measurements inside vegetated and urban canopies provided the Lagrangian time scale (or $C_0$) is properly parameterized (Hsieh et al., 2003; Molder et al., 2004; Raupach, 1987, 1989a, b; Rotach et al., 2004). These agreements are suggestive that the basic LDM, especially those based on the widely used Thomson’s simplest solution (Thomson, 1987; Wilson and Sawford, 1996), do capture the essential scalar dispersion processes inside canopies, and the effects of Reynolds number or the absence of an ISR can be accounted for by some ‘zeroth-order’ modification to $C_0$. Hence, the main novelties of this work are not to provide new theories for $C_0$ or revise existing LDM formulations (especially the dispersion term). Rather, we start by assuming that standard LDM formulations are not to be altered within the CSL and estimate how $C_0$ relative to the free atmospheric state should be optimally altered to account for the absence of an ISR and finite Reynolds. It is assumed throughout that the profiles of the flow statistics, including the mean velocity, the components of the Reynolds stress tensor, and the mean turbulent kinetic energy dissipation rate are all known or can be independently inferred (e.g. from Eulerian higher order closure models).

Towards this goal, detailed laser Doppler anemometry (LDA) measurements were collected in a flume within and above a densely arrayed canopy composed of steel rods. These velocity measurements were used to compute first and second moments, the mean turbulent kinetic energy and its dissipation rate, the integral time scales, and $R_z$. What distinguishes these experiments when compared to various field experiments, reviewed in Table 2, is that the energy spectrum here unambiguously exhibits both a short-circuiting region and a wake production region. Most field experiments to date reported some short-circuiting in the energy cascade, but because of their usage of sonic anemometry, wake production is rarely resolved. Typical averaging path length of commercial sonic anemometers is on the order of 10 cm, which is comparable to the order of magnitude of wake production length scales (branches and small stems). These field experiments cannot unambiguously discern a wake production region given the multiplicity of length scales in real canopies, which is in stark contrast to wind tunnel and flume experiments that usually employ single diameter rods (see Table 2).

The effects of finite $R_z$ were explored using the recent formulations in Lien and D’Asaro (2002). These formulations, which explicitly show how $C_0$ varies with $R_z$, were derived by matching a wide range of DNS results and laboratory data sets and generally agree with other formulations (Lien and D’Asaro, 2002; Ouellette et al., 2006a, b; Sawford, 1991; Yeung et al., 2006). To compute the effective $C_0$ variations inside the canopy, estimates of the Lagrangian time scales from their Eulerian counterpart is necessary, though such estimates remain problematic. Classical formulations assume that the

<table>
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<tr>
<th>Study</th>
<th>Short-circuiting</th>
<th>Wake production</th>
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<tbody>
<tr>
<td>Wind tunnel and flume experiments</td>
<td></td>
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<tr>
<td>Seginer et al. (1976)</td>
<td>Yes</td>
<td>Yes, at $St = 0.21$</td>
</tr>
<tr>
<td>Raupach et al. (1986)</td>
<td>Yes</td>
<td>Yes, for $w$ (including data just above the canopy)</td>
</tr>
<tr>
<td>Poggi et al. (2004c)</td>
<td>Yes</td>
<td>Yes, at $St = 0.21$</td>
</tr>
<tr>
<td>Atmospheric surface layer/tower wake</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barthlott and Fiedler (2003)</td>
<td>Yes</td>
<td>Yes, mainly for $w$ and $v$. They note that the frequency of tower wakes is independent of stability</td>
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<tr>
<td>Forested canopies</td>
<td></td>
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<tr>
<td>Amiro (1990)</td>
<td>Yes</td>
<td>No, except a secondary peak in $w$ was observed at the lowest level in a black-spruce forest canopy</td>
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<tr>
<td>Baldocchi and Meyers (1988)</td>
<td>Yes</td>
<td></td>
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<tr>
<td>Blanken et al. (1998)</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>Gardiner (1994)</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>Krujit et al. (2000)</td>
<td>Yes</td>
<td>No, except for a possible value at $z/h = 0.27$ at Cuieiras forest</td>
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<td>Liu et al. (2001)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Villani et al. (2003)</td>
<td>Yes</td>
<td>No, except for stable runs</td>
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Note the limited number of field studies resolving wake production. $St$ is the Strouhal number defined in the text.
ratio of the Lagrangian to Eulerian time scales varies with turbulent intensity (Anfossi et al., 2006; Angell et al., 1971; Corrsin, 1963; Degrazia and Anfossi, 1998; Hanna, 1981; Philip, 1967; Rizza et al., 2006). Here, these results are expanded to include relationships recently derived from a wide range of data sets including field experiments and large eddy simulation (LES) runs.

The absence of an ISR on $C_0$ is explored by noting how the short-circuiting of the energy cascade and wake production ($W_p$) alter the Eulerian ISR. Simplified theories are then used to scale these Eulerian modifications to possible modifications in $C_0$. For the short-circuiting of the energy cascade, a simplified spectral model recently proposed by Finnigan (2000) is employed for a first-order analysis. This model can be reformulated as a linearized ‘correction’ to the Eulerian similarity constant of the velocity spectrum ($\sigma_u$). Using recent theoretical relationships between $C_0$ and $\sigma_u$ (Franzese and Cassiani, 2007), estimates of the $C_0$ adjustments are then derived. The effect of $W_p$ on $C_0$ is difficult to theoretically derive because wake production introduces energy-backscatter and possibly an inverse energy cascade. However, the data collected here provide some qualitative patterns showing how computed $C_0$ might vary with $W_p$ but the generalities of these results remain to be tested.

Throughout, the longitudinal, lateral, and vertical directions are designated as $x_1$ or $x$, $x_2$ or $y$, $x_3$ or $z$, respectively, and the concomitant velocities along these directions are $u_1$ or $u$, $u_2$ or $v$, and $u_3$ or $w$, over-bar denotes time averaging, $\langle \cdot \rangle$ denotes planar averaging (Raupach and Shaw, 1982), and $t$ is time, primed quantities are excursions from the mean turbulent kinetic energy dissipation rate, and $d\Omega_j$ are the directional Wiener increments (i.e. a white-noise Gaussian process with zero mean and variance $d\tau$) along $x_j$. The numerical value of $C_0$ remains the subject of debate even in idealized flows with values reported anywhere from 2 to 7. A partial summary of these values of $C_0$ determined from a number of sources, experiments, and methods is presented in Table 1.

The value of $C_0$ can be related to the Lagrangian integral time scale, $T_L$, by assuming that Eq. (1) describes the entire structure function, integrating Eq. (1) from an upper limit bounded by $T_L$ to a lower limit bounded by the viscous dissipation scale, $T_\eta$, and upon ignoring the latter with the respect to the former and assuming that this range of time scales capture the entire velocity variance ($\sigma^2$) yields

$$2\sigma^2 = C_0 \varepsilon \int_{T_\eta}^{T_L} d\tau'$$

so that

$$T_L = \frac{2\sigma^2}{C_0 \varepsilon}.$$  

Hence, the value of $C_0$ may be inferred in the so-called equilibrium layer, where production of turbulent kinetic energy (TKE) balances $\varepsilon$ (resulting in $\varepsilon = u_t^3(kz)^{-1}$, where $u_t$ is the friction velocity), by either assuming that (i) $T_L$ can be described from similarity theory arguments ($T_L \sim kz/u_t$) or (ii) by matching the Lagrangian and Eulerian turbulent diffusivities ($K_t$).

In the case of the vertical velocity component ($w'$), the former leads to

$$C_0 = \frac{2\sigma_w^2}{(kz/u_t)(k^3/k^2)} = 2\left(\frac{\sigma_w}{u_t}\right)^2,$$  

while the latter leads to

$$K_t = \sigma_w^2 T_L = \frac{2\sigma_w^4}{C_0 \varepsilon} = kz u_t$$  

and with $\varepsilon = u_t^3(kz)^{-1}$ yields

$$C_0 = 2\left(\frac{\sigma_w}{u_t}\right)^4.$$  

Within the equilibrium region of a neutral boundary layer, $\sigma_w/u_t \approx 1.25$ resulting in $C_0 \approx 3.125$ when matching the Lagrangian time scale to similarity theory (Li and Taylor, 2005) and a $C_0 \approx 4.9$ when matching the turbulent diffusivities (Rodean, 1991).

2. Scaling analysis

The dispersion term in standard LDM formulations ($= \sqrt{C_0 \varepsilon d\Omega_j}$) assumes that the second-order Lagrangian structure function $D(\Delta t)$ in the ISR is given by (Monin and Yaglom, 1975)

$$D(\Delta t) = C_0 \varepsilon \Delta t,$$  

where $C_0$ is the Lagrangian–Kolmogorov constant, $\Delta t$ is the time separation that is much smaller than the integral time scale but much larger than the Kolmogorov viscous dissipation time scale, $\varepsilon$ is the
Another estimate of $C_0$ may be derived by setting $T_L = \beta T_E$ (Corrsin, 1963; Hay and Pasquill, 1959), where $T_E$ is the measured Eulerian integral time scale, and $\beta$ is a proportionality constant. For the $u'$ component (any component can be used), let $T_E = I_u$, the integral time scale of the longitudinal, so that

$$C_0 = \frac{2\sigma_u^2}{\beta \overline{u}'}. \quad (6)$$

The $I_u$ can be determined from Eulerian autocorrelation function measurements, given by

$$\rho_u(t') = \frac{\overline{u'}(t)\overline{u'}(t + t')}{\overline{u'^2}(t)}.$$

Fig. 2 shows the canopy drag using

$$I_u = \int_0^\infty \rho_u(t') \, dt',$$

where the integration is often terminated at the first zero-crossing. However, the value of $\beta$ is not a priori known. Corrsin (1963) pointed out that ‘since the Eulerian autocorrelation involves new fluid continuously wandering past the observation point, while the Lagrangian one follows a material point, we expect the latter to be more persistent’. Or, stated differently, the Lagrangian time scale should be larger than its Eulerian counterpart so that $\beta > 1$. Hence, if $\beta = 1$, $C_0$ will be at its maximum possible value.

To derive an expression for $\beta$, Corrsin (1963) noted that the Eulerian and Lagrangian velocity spectra ($m^2 s^{-1}$) in the ISR (unperturbed by the presence of a canopy) may be expressed as

$$E_E(n) = \overline{u'^2}^{1/3} \overline{u'}^{2/3} n^{-5/3},$$

$$E_L(n) = C_0 n^{-2}, \quad (9)$$

where $n$ is frequency ($= 1/\Delta t$), and $\overline{u'} \approx 0.55$ is, as before, the Kolmogorov constant for the longitudinal Eulerian spectrum, and $\overline{U_c}$ is the convective velocity of eddies. Assuming these power-law spectra represent the entire velocity spectra, and upon integrating these two spectra from frequencies corresponding to the integral time scale up to frequencies corresponding to the Kolmogorov viscous dissipation time scales (assumed much smaller than the integral time scales), and matching the outcome of these two integrals (given the variance equality of $\sigma^2$), we obtain:

$$2\sigma^2 \approx 3/2 \overline{U_c'^2} \overline{u'}^{2/3} T_E^{2/3},$$

$$2\sigma^2 \approx C_0 \overline{u} T_L.$$

Hence, from these two expressions,

$$\frac{T_L}{T_E} = \beta = \left[ \frac{\left( \frac{3}{2} \overline{u'}^{2/3} / C_0 \right)^{3/2}}{\frac{1}{i}^{1/3} \gamma} \right]_{1},$$

where $i = \sigma / \overline{U_c} \approx \sigma / \overline{U}$ is the turbulent intensity. Here, it is assumed that the mean velocity approximates the convective velocity of eddies (i.e. $\overline{U} \approx \overline{U_c}$), a reasonable assumption only in low intensity flows ($\bar{f} < 0.1$) as discussed elsewhere (Hsieh and Katul, 1997).

The relationship between $\beta$ and $i$ has been studied by a number of authors (Pasquill, 1974), and was recently revisited by Anfossi et al. (2006) via LES experiments in which both the Lagrangian and Eulerian autocorrelation functions were reported. These LES runs suggest that $\beta \approx \gamma(1/i)$ predicts $T_L/T_E$ reasonably well (see Fig. 1) for a wide range of atmospheric stability conditions. When combining all their LES runs for the three velocity components, Anfossi et al. (2006) obtained a value for $\gamma = 0.41$ (see Fig. 1). These results are in agreement with a large number of experimental studies and scaling analysis, also summarized in Fig. 1, especially from Wandel and Kofoed-Hansen (1962) who found $\gamma \approx 0.55$. Wang et al. (1995) also used an LES for a channel flow for two different Reynolds numbers (varied by a factor of 7) and found that when all the data for the three velocity components were combined, $\gamma \approx 0.6$. These two LES experiments appear consistent with the classical balloon trajectory experiments of Hanna (1981), who found that $0.35 < \gamma < 0.8$. Note that when $\gamma = 0.41$ and $\overline{u'} = 0.55$, $C_0 = 1.83$ if Corrsin’s (1963) arguments are used. Some authors (Koeltzsch, 1999) correctly argued that $\gamma$ cannot be constant because $\overline{U}/\overline{U_c}$ need not be unity and can vary with the flow configuration (e.g. roughness type). Because of these uncertainties in $\gamma$, all $C_0$ estimates inside the canopy will be reported relative to the $C_0$ values well above the canopy thereby minimizing the sensitivity of its attenuation to the precise $\gamma$ estimate.

### 2.1. Reynolds number effects on $C_0$

The Lagrangian structure function, $D(n) = C_0 \Delta n^{-1}$, is derived for infinite Reynolds number and estimates of $C_0$ from various experiments are affected by finite Reynolds number effects. Over the past two decades, these effects have been studied extensively via scaling analysis, DNS, and laboratory experiments (Heinz,
Lien and D’Asaro (2002) recently proposed an interpolation function that matches a wide range of DNS and laboratory data sets, given by

\[ C_0 / C_0^* = \begin{cases} 
1 - \frac{1}{\sqrt{0.1 R_l}}, & R_l > 100, \\
0.068 \sqrt{R_l}, & R_l < 100,
\end{cases} \]

where \( R_l \) is the Taylor micro-scale Reynolds number, given by

\[ R_l = \frac{\sigma_\lambda}{v}, \quad \lambda = \sigma_u \sqrt{\frac{15}{\varepsilon}}, \]

and \( C_0^* \) is the value of \( C_0 \) inferred at infinite \( R_l \).

In the equilibrium layer of the atmosphere (\( v = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \)), at a typical \( z = 5 \text{ m} \) and for a typical daytime \( u_* = 0.3 \text{ m s}^{-1} \) results in \( \varepsilon = 0.0135 \text{ m}^2 \text{ s}^{-1}, \sigma_\lambda = 0.8 \text{ m s}^{-1}, \lambda = 0.1 \text{ m}, R_l \approx 5, 300, \) and \( C_0 / C_0^* \approx 0.96. \) However, inside the canopy, \( \varepsilon \) may increase by a factor of 10 above its free atmospheric value and \( \sigma_\lambda \) may be attenuated by a factor of 2, resulting in \( \lambda = 0.016 \text{ m}, R_l \approx 435, \) and a \( C_0 / C_0^* \approx 0.83, \) not a trivial reduction over its free atmospheric value.

2.2. The effects of short-circuiting of the energy cascade on \( C_0 \)

Fig. 2 shows how the canopy drag \( (C_d a) \) modifies the Eulerian spectrum (commensurate with ISR range of scales) inside a dense canopy. Here \( C_d \) is the drag coefficient and \( a \) is the rod density in \( \text{m}^2 \text{ m}^{-3}; \) in vegetated canopies, it is similar to the leaf area density.

The Eulerian spectrum now includes a wake production region and a short-circuiting of the energy cascade. To explore the effects of short-circuiting, the spectral model of Finnigan (2000) is used as a starting point (hereafter referred to as F00). F00 proposed a simplified model to account for the short-circuiting of the energy cascade due to the action of the canopy drag on turbulence. In essence, this model assumes that in addition to the flow of energy from wavenumber \( k \) to wavenumber \( k + dk \), there is an extra energy sink proportional to \( C_d a U \). For the inertial subrange scales, the resulting equation governing the energy spectrum is given by

\[ \frac{d}{dk} (z u_1^{-1/3} k^{5/3} E(k)) = -\frac{3}{2} C_d a U E(k), \]
and whose solution is

$$E(k) = z_0 \left[ \exp \left( \frac{3}{2} z_0 C_{da} \bar{U} \varepsilon^{-1/3} k^{-2/3} \right) \right] \varepsilon^{2/3} k^{-5/3}$$

$$= [z_0'] \varepsilon^{2/3} k^{-5/3}.$$  

The integration constant for this differential equation was evaluated by noting that when $C_{da} \to 0$, the Kolmogorov ISR must be recovered. Note that this adjustment involves two characteristic time scales: the 'adjustment frequency' $C_{da} \bar{U}$ (formed by
combining the adjustment length scale \( L_c = (C_d a)^{-1} \) and a local mean velocity) and a local relaxation time scale \((dk^3)^{-1/3}\) that depends on the local wavenumber. The optimum \( \zeta_u' \) can be chosen to preserve the area under the two spectral models within the energy-short-circuiting domain, resulting in

\[
\zeta_u' = \int_{k=1/k}^{k=1/d} \exp\left(\frac{9}{4} k a C_d a \bar{U} \varepsilon^{-1/3} k^{-2/3}\right) \tilde{C}^{2/3} k^{-5/3} \, dk
\]

Fig. 3 illustrates why \( \zeta_u' \) can increase due to the short-circuiting of the energy cascade using typical flow variables values from a 23-year-old pine stand. In terrestrial ecosystems, \( \zeta_u'/\zeta_u \) can be large, up to a factor of 2, though for the flume experiments, it will be smaller as we show later.

At time increments commensurate with LDM integration time steps, the short-circuiting of the energy cascade is likely to become small when compared to \( W_p \), which occurs at wavenumbers commonly estimated from the Strouhal frequency \( f_v \) given by

\[
f_v = 0.21 \frac{\bar{U}}{d}, \quad k_v = \frac{0.21}{d},
\]

where \( d \) is an effective diameter (or rod diameter in the case of cylindrical morphology). At these wavenumbers, the wake production rate is on the order of \( W_p \approx \bar{U}^3/L_c \). When considering the energy cascade from \( k \) to \( k + dk \) in the vicinity of \( k_v \), a fraction of \( W_p \) is actually injected not removed from the spectrum (Fig. 2), unlike the short-circuiting phenomenon. Hence, in this case, \( C_0 \) must decrease to compensate for this new source of energy in the spectrum (i.e. \( k_v \)). To take this effect into account, a simplified spectral net dissipation model is proposed next.

2.3. Wake production effects on \( C_0 \)

The classical spectral view of the ISR is that this region experiences a constant net flux of energy \((= \varepsilon)\) with no energy being added by the mean flow or subtracted by the action of viscosity. Hence, this net dissipation rate does not vary with wavenumber in the classical ISR (beyond the usual intermittency corrections). Velocity spectra, reported elsewhere (Poggi and Katul, 2006; Poggi et al., 2004c), already demonstrated that wake production is centered around \( f_v \) (see Table 2) and this production can be large (when compared to \( \varepsilon \)). These studies also demonstrated that an appreciable ‘spectral broadening’ around this value exist. Currently, formulations analogous to F00 that predict the shape of the spectrum from such an injection is lacking, and hence, an analytical treatment remains illusive.

A model for the net flux of energy as a function of wavenumber (rather than the spectrum) may be formulated when considering the net production and dissipation as shown in Fig. 2 (bottom panel). The classical ISR here is represented as \( \varepsilon_0 \) being constant with \( k \), the short-circuiting effects previously discussed tend to introduce an exponential behavior with \( k \) as proposed by F00 \( (\varepsilon'_0 \) as shown in Fig. 2). This effect, as we show later, is not as significant as the wake production for the flume experiments. Hence, for simplicity, we assumed a constant \( \varepsilon \) \((= \varepsilon_0)\) up to the wake injection scales. At these injection scales, net energy from wakes is concentrated around a typical length scale defined as \( L_w = k_v^{-1} \) (i.e. the broadening is neglected as well). With this primitive representation of the scale-wise dissipation rate, the equivalent (i.e. scale-wise area-matching) dissipation that should be used in LDM, \( \tilde{\varepsilon} \), is given by

\[
\tilde{\varepsilon} = \int_{L_v^{-1}}^{\eta^{-1}} \varepsilon(k) \, dk \left(\eta^{-1} - L_v^{-1}\right)^{-1}
\]

\[
\approx \varepsilon_0 \left(\frac{L_w^{-1} - L_v^{-1}}{\eta^{-1} - L_v^{-1}}\right) + W_p \left(\frac{\eta^{-1} - L_w^{-1}}{\eta^{-1} - L_v^{-1}}\right)
\]

\[
= \varepsilon_0 + W_p \frac{\eta^{-1} - L_w^{-1}}{\eta^{-1} - L_v^{-1}},
\]
where \( \eta = (v^3/\nu)^{1/4} \) is, as before, the Kolmogorov micro-scale and \( L_t \) is the macro-scale at which shear production injects energy into the cascade. If \( L_t \gg L_w \), then
\[
\bar{\varepsilon} = \varepsilon_0 + W_p \left(1 - \frac{\eta}{L_w}\right).
\]

Hence, for LDM applications, the optimum \( C_0\bar{\varepsilon} = C_0\varepsilon_0 \), where \( \varepsilon_m \) is the measured dissipation energy rate, results in \( C_0/C_0' = \varepsilon_m/\bar{\varepsilon} \) so that
\[
\frac{C_0}{C_0'} = \frac{\varepsilon_m - W_p + W_p(1 - \eta/L_w)}{\varepsilon_m} = 1 - \frac{W_p}{\varepsilon_m} \frac{\eta}{L_w}.
\]

The ‘correction’ to \( C_0 (= C_0') \) is due to two dimensionless groups: \( W_p/\varepsilon_m \) and \( \eta/L_w \). The first represents the ratio between the energy flux injected into the cascade through wake production and the total dissipation rate, and the second is the ratio between the length scale at which the dissipation takes place and the length scale around which \( W_p \) is injected. From this model, a number of hypotheses about the magnitude of this \( C_0 \) correction can be formulated:

1. When wake production is much smaller than the flux of energy injected into the cascade from the shear production, the correction to \( C_0 \) is negligible. Close to the ground, the shear production is negligible and \( W_p \) may be the only leading term balancing \( \bar{\varepsilon} \) in the turbulent kinetic energy balance. In this case, \( W_p/\varepsilon_m \to 1 \) and \( C_0'/C_0 \to 0 \).
2. When \( W_p \) is injected at very small scales, say comparable with \( \eta, \eta/L_w \to 1 \) and the ratio \( C_0'/C_0 \to 0 \).
3. When \( W_p \) is injected at scales much larger than \( \eta, \eta/L_w \to 0 \) and \( C_0'/C_0 \to 1 \), and the correction is, again, negligible.

Using these scaling arguments, the experiments described next are aimed at quantifying how much the reductions or increases in \( C_0 \) are due to \( R_s \), wake production, and energy short-circuiting.

3. Experiment

While much of the experimental setup, sensors used, and data processing has been presented elsewhere (Poggi et al., 2002, 2003, 2004a–d, 2006; Poggi and Katul, 2006), the salient features are briefly reviewed. The within-canopy velocity measurements were conducted at the hydraulics Laboratory, DITIC Politecnico di Torino, in a large re-circulating constant head channel. The channel dimensions are 18 m long, 0.90 m wide, and 1 m deep with glass sidewalls to permit optical access. The working test section is 9 m long, 0.9 m wide, and begins at 7 m downstream from the main water entrance.

Two experiments were carried out at different flow rates resulting in a friction velocity at the canopy top of \( u_* = 0.053 \) and \( u_* = 0.098 \text{ m s}^{-1} \). The canopy configuration, described next, was not altered across these two experiments. The canopy Reynolds number, \( Re_c = u_* h / \nu \) varied from 6360 (hereafter referred to as the lower Reynolds number experiment) to 11,760 (hereafter referred to as the higher Reynolds number experiment), which is about one to two orders of magnitude lower than what is expected for canopy flows inside forested ecosystems. In forested systems, typical \( u_* = 0.3 \text{ m s}^{-1} \) and typical \( h = 20 \text{ m} \) resulting in an atmospheric canopy \( Re_c = 400,000 \). However, \( Re_c \), as we show later, are more comparable between these experiments and CSL flows in forested and agricultural ecosystems.

3.1. Canopy configuration

The canopy was composed of an array of vertical steel cylinders, 12 cm tall \( (h) \) and 4 mm in diameter \( (d) \), arranged in an aligned pattern across the entire test section. The rod density was 1072 rods m\(^{-2} \), dynamically equivalent to a dense canopy with a frontal area index of 4.2 m\(^2\) m\(^{-3} \). During the experiments, the water depth \( (h_w) \) was steadily maintained at 60 cm.

3.2. Velocity measurements

The velocity statistics were measured using a two-component LDA in forward scattering mode. The sampling duration and frequency were 3600 s and 2500–3000 Hz, respectively. About 0.6\( h_w \) was sampled at 1 cm vertical increments at planar locations described elsewhere.

3.3. TKE dissipation rate profile estimates

The mean turbulent kinetic energy dissipation rate was estimated from the isotropic relationship (and Taylor’s frozen turbulence hypothesis),
\[
\bar{\varepsilon} = \varepsilon_m = 15 \nu \frac{1}{U^2} \left( \frac{\partial u}{\partial t} \right)^2,
\]
and is hereafter referred to as ‘measured’ dissipation rate (see Fig. 2, bottom panel) to distinguish it from other estimates. These \( e_m \) estimates agree well with independent \( \varepsilon \) estimates derived from velocity spectra (hereafter referred to as \( e_{\text{spec}} \)) measured above the canopy assuming \( z_u = 0.55 \) (see Appendix for statistical comparisons). Inside the canopy, it was demonstrated in another study that \( e_m \) estimated from the above isotropic relationship agrees well with \( \varepsilon \) estimated from the residual of the turbulent kinetic energy budget (Poggi et al., 2006), where all other terms (mainly wake and shear production and flux transport terms) are independently measured or estimated. These turbulent kinetic energy dissipation rate profiles are ‘externally specified’ to all \( C_0 \) (and \( R_c \)) calculations.

When computing \( \varepsilon \) by fitting the measured energy spectrum to \( E_u = z_u e^{2/3} k^{-5/3} \) for scales smaller than \( h \) but larger than \( d (e_{\text{spec}}) \) inside the canopy (there is really no ISR as in Figs. 2 and 3), adjustments to \( z_u \) (and as we show later \( C_0 \)) can also be derived. For example, in Fig. 2, if the modeled \( E_u = z_u (e_{\text{spec}})^{2/3} k^{-5/3} \) was forced to match the measured \( E_u \) but retaining \( z_u = 0.55 \) (i.e. commensurate with some constant reference \( C_{0,\text{ref}} \) expected for the free atmosphere well above the canopy), then \( C_0 \) can be empirically determined from

\[
\frac{C_0}{C_{0,\text{ref}}} = \frac{e_{\text{spec}}}{e_m}.
\]

These dissipation ratio based \( C_0 \) adjustments inside the canopy can be used to assess whether the computed \( C_0 = 2\sigma_u^2/(\beta l_w e_m) \) inside the canopy are reasonable. Note that this approach does not assume that the inertial subrange power-law scaling extends throughout the entire spectrum and explains the entire \( \sigma^2 \), as was done in the earlier derivation. Rather, it provides an empirical estimate of the best \( C_0 \) that recovers the area under the measured energy spectrum within the ISR if K41 scaling is used and if \( \varepsilon \) is a priori specified (see Fig. 2, top panel).

3.4. Comparison with CSL field experiments

To assess how well this experiment represents dense agricultural and forested ecosystems, Fig. 4 shows a comparison between measured flow statistics from these two flume experiments and reported measurements from a wide range of terrestrial canopies reviewed elsewhere (Katul et al., 2004). By and large, these flume experiments are characterized by a CSL exhibiting a faster \( \bar{\nu}/u_\ast \), less attenuated mean momentum fluxes \( \bar{\nu}^2/\bar{u}_\ast^3 \), and a lower \( \sigma_u/u_\ast \). The \( \varepsilon \) is rarely reported in field experiments, but qualitatively the flume experiments here appear consistent with wind tunnel experiments presented in Raupach (1988), though the peak value is about 20% lower.

In terms of \( R_c \), the lower Reynolds number experiment has, as expected, a lower normalized \( \bar{\nu}/u_\ast \), comparable \( \bar{\nu}^2/\bar{u}_\ast^3 \), a lower \( \sigma_u/u_\ast \), and a lower normalized \( \varepsilon h/\bar{u}_\ast^3 \) when compared to the higher Reynolds number experiment. The spread in \( \varepsilon h/\bar{u}_\ast^3 \) is likely to generate ample spread in \( R_c \) as well.

4. Results

Estimates of \( C_0 \) and their potential variation with \( R_c \) and \( W_p \) are presented here. To compute \( C_0 \) from Eq. (6), \( T_L \) (or \( \beta \)) is required. Fig. 5 shows the variation of the key time scales within the CSL: normalized \( I_u \) and \( I_w \), \( T_L = I_u(0.41 \bar{h}^{-1}) \), and the so-called relaxation time scale \( \tau = \text{TKE}/\varepsilon \). In simplified LDM formulations, a constant \( T_L \) within the canopy is often used and is also shown in Fig. 5 for reference (Raupach, 1988). The flume data here suggest that \( I_w \) is at least two times smaller than \( I_u \), and the variations in \( T_L \) (based on \( I_u \)) better agree with Raupach’s (1988) suggestion. Hence, estimates for \( C_0 \) will be based on the \( u \) component rather than the \( w \) component for this reason. While \( I_w \) and \( I_u \) appear not to be sensitive to \( R_c \), the \( \tau \) does exhibit a significant Reynolds number dependence. The \( \tau \) profile demonstrates that the increase in TKE does not compensate for the increase in \( \varepsilon \) with increasing \( R_c \). However, for the Reynolds number dependence of \( C_0 \), it is \( R_c \) not \( R_{C_0} \) that is relevant.

4.1. Finite Taylor micro-scale Reynolds number effects

Fig. 6 shows \( R_c(z) \) inside the canopy, and the computed reductions in \( C_0 \) (Lien and D’Asaro, 2002). In the lowest layers of the canopy, these calculations suggest that \( C_0 \) may be reduced by as much as 50% only due to the finite Reynolds number effects. Near the canopy top and aloft, these reductions become on the order of 10% and can be neglected in a first-order analysis.

Fig. 6 also shows the computed \( C_0 \) from Eq. (2) using \( \sigma = \sigma_u \) and \( T_L = I_u \bar{h}^{-1} \). As earlier stated, to ensure that the results are less sensitive to choice
of $\gamma$, the $C_0$ profile is normalized by the mean $C_0$ well above the canopy ($z/h > 1.25$). Hence, if $\gamma$ is constant for each of the two experiments, then the $C_0$ profiles in Fig. 6 can be interpreted as ‘attenuation’ profiles due to the presence of the canopy. It is clear that the attenuation of $C_0$ is more severe (factor of 5) than what can be explained by finite Reynolds number effects (at most a factor of 1.5).

Fig. 7 shows the estimates of $C_0(z)$ using

$$\frac{C_0(z)}{C_{0,\text{ref}}} = \frac{\varepsilon_{\text{spec}}}{\varepsilon_{\text{m}}}$$

where, as before, $\varepsilon_{\text{spec}}$ was estimated by fitting the measured Eulerian energy spectrum to K41 assuming $\alpha_u = 0.55$ (i.e. the reference $u$). The variations $\varepsilon_{\text{spec}}/\varepsilon_{\text{m}}$ are in good agreement with the $C_0 = 2\sigma_u^2/\langle\beta u'v'\rangle$ when normalized by the mean value in the surface layer. Hence, this analysis can be treated as an empirical verification that the modifications to the Eulerian spectrum by the canopy reduce $C_0$. How much of this ‘added’ attenuation is due to short-circuiting or wake production is explored next.

4.2. Short-circuiting of the energy cascade effects

As earlier noted, for the F00 spectrum at high wavenumbers, the effects of short-circuiting of the
Fig. 5. The vertical variation of the Eulerian integral time scales (longitudinal $I_u$, vertical $I_w$) for the two flume experiments. The computed Lagrangian integral time scale using $T_L = 0.41 I_u^\kappa$ is also shown (middle) along with the proposed Lagrangian time scale model of Raupach (1988) (solid line). For reference, the relaxation time scale $\tau = \text{TKE}/\varepsilon$ is shown (right). The length and time scales are all normalized by $h$ and $h/u_*$, respectively.

Fig. 6. Vertical variation of the Taylor micro-scale Reynolds number (left), the expected adjustment to $C_0$ from the formulation of Lien and D’Asaro (2002) (middle), and $C_0 = 2\sigma_0^2 / 3I_u^\kappa$ but normalized by the mean values of $C_0$ above the canopy thereby making these profiles independent of $\gamma$ (right).
energy cascade over its ISR value can be lumped into an effective dimensionless parameter $\gamma_u$ given as

$$\frac{\gamma'_u}{\gamma_u} = \frac{\int_{k=1/h}^{k=+1/d} [\exp((9/4)\gamma_u C_0 a d U_e^{-1/3} k^{-2/3})] k^{2/3} k^{-5/3} dk}{\int_{k=1/h}^{k=+1/d} e^{2/3} k^{-5/3} dk}$$

$$\approx \frac{\int_{k=1/h}^{k=+1/d} (1 + (9/4)\gamma_u C_0 a d U_e^{-1/3} k^{-2/3}) e^{2/3} k^{-5/3} dk}{\int_{k=1/h}^{k=+1/d} e^{2/3} k^{-5/3} dk}$$

$$\approx 1 + \frac{9}{8} \gamma_u C_0 a d U_e^{-1/3} h^{2/3}$$

and leads to an (analytical) adjustment in $C_0$ estimated as (Franzese and Cassiani, 2007),

$$\frac{C_0}{C_{0,ref}} = \left[ \frac{\gamma'_u}{\gamma_u} \right]^{3/2} \approx \left( 1 + \frac{9}{8} \gamma_u C_0 a d U_e^{-1/3} h^{2/3} \right)^{3/2}$$

where the exponential term was linearized and $d \ll h$. Naturally, outside the canopy, $C_d = 0$ and $C_0 = C_{0,ref}$. For order-of-magnitude analysis, assume that $C_d a \approx 0.5 \text{m}^{-1}$, $\gamma_u \approx 0.55$, $\varepsilon \approx 5(u_0^3/h)$, $U \approx 3.3u_a$ (see Fig. 4 for representative upper canopy values), $h = 0.12 \text{m}$ to yield

$$\frac{C_0}{C_{0,ref}} = \left[ \frac{\gamma'_u}{\gamma_u} \right]^{3/2} \approx \left( 1 + \frac{9}{8} 0.55 \times 0.5 \times 3.3 \times 0.12 \right)^{3/2} \approx 1.11.$$
that impacts \( \frac{C_0}{C_0} \) that is absent from any free-atmospheric considerations, where \( C_0 \) may be a constant. This integrated model must be constructed to satisfy the following constraints:

1. recover the constant \( C_0 \) as \( W_p - 0 \);
2. model vertical changes in local \( C_0 \) to be proportional to the local \( W_p \);
3. \( W_p \) only varies with \( z \) (as is the case here).

Using these three arguments, a plausible model for the attenuation of \( C_0 \) is

\[
\frac{dC_0}{dz} = A(W_p),
\]

where \( A \) is a constant. Here, the assumption is that \( C_0 \) varies from one level to another primarily due to the local strength of \( W_p \). Assuming \( W_p \approx \bar{U}^3/L_c \) and \( \bar{U} \sim U_h \exp(-\beta z') \), and using a constant \( L_c \),

\[
\frac{dC_0}{dz} = A \left[ \frac{U_h^3}{L_c} \exp(-3\beta' z') \right],
\]

which upon integration yields

\[
\frac{C_0(z)}{C_{0,b}} \sim \exp(-3\beta'(z')),\]

where \( \beta' \) is the mean velocity attenuation coefficient, \( z' = 1-z/h \) and \( C_{0,b} \) is the value of \( C_0 \) at the canopy top. Fig. 8 shows that when \( \bar{U}/U_h \sim \exp(-z') \), \( C_0/C_{0,b} \sim \exp(-3z') \) suggestive that \( C_0 \) inside the canopy is reduced primarily because of wake production. This ‘exponential model’ also captures much of the reductions in \( C_0 \) only using the attenuation in the mean velocity profile, and can be used in operational models in lieu of \( \frac{C_0}{C_0} = 1 - (W_p/\bar{U}_m)(\bar{U}/L_W) \). However, its generality needs further exploration.

5. Discussion

A number of studies have already shown that \( C_0 \) may not be universal when the flow significantly diverges from homogeneous isotropic stationary turbulence (HIST). For example, Heinz (2002) argued that in a generalized Langevin model, the velocity fluctuations can be decomposed into three canonical contributions—an anisotropic contribution due to gradients in the Eulerian flow statistics (appearing in the drift term of LDM), a Lagrangian acceleration fluctuation term, and a stochastic forcing. High values of \( C_0^* \) are controlled by the disappearance of the first two effects (Heinz, 2002). When the effects of anisotropy and acceleration fluctuations contribute to the energy budget, \( C_0^* \) values are generally low (~1.0–3.0), and conversely, a value of \( C_0^* = 6 \) is approached when these two effects disappear (e.g. HIST).

Inside canopies, large anisotropy and local acceleration from wakes are expected, and hence, a reduction in \( C_0^* \) (and subsequently \( C_0 \)) is in qualitative agreement with Heinz’s (2002) analysis. Furthermore, the phenomenological model proposed here to reducing \( C_0 \) with \( z' \) inside the canopy being proportional to the main source of anisotropy and local acceleration—\( W_p \) is also qualitatively consistent with Heinz’s (2002) analysis.
A logical follow-up question to ask is whether such vertical variations in \( C_0 \) inside canopies are important to LDM predicted concentration. The sensitivity of LDM models to the absolute value of \( C_0 \) (or surrogates for it) inside urban and vegetated canopies has already been demonstrated. For predicting mean concentrations inside urban canopies using LDM, Rotach et al. (2004) underlined their sensitivity to the magnitude of \( C_0 \) and the shape of the \( \varepsilon \) profile. Likewise, Molder et al. (2004) and Rannik et al. (2003) also showed that LDM results are sensitive to the assumed shape of the Lagrangian integral time scale (connected to \( C_0 \)). No study to date, however, has evaluated the sensitivity of the mean concentration profile to possible variations in \( C_0 \) inside the canopy. Towards this end, Fig. 9 presents LDM concentration calculations for two source releases, \( z/h = 0.2 \) and 0.8, using the Eulerian velocity statistics shown in Fig. 1 (for the lower \( Re_c \)), and using \( C_0/C_{ref} = (-3z') \) inside the canopy as shown in Fig. 8 but keeping \( C_0/C_{ref} = 1 \) above. These LDM calculations for the variable and constant \( C_0 \) are compared with calculations that assume \( C_0/C_{ref} = 1 \) for the entire domain (\( C_{ref} = 6 \) is chosen for reference, but the structural differences in mean concentration profiles between the constant and variable \( C_0 \) are not sensitive to \( C_{ref} \)).

To facilitate the comparison, we focus on two regions—near the source at \( x/h = 1, 2 \) and far from the sources at \( x/h = 10 \). The three-dimensional LDM used in these calculations is based on Thomson’s (1987) simplest solution and is described elsewhere (Poggi et al., 2006). What the model calculations suggest is that for source releases near the canopy top, the effect of variable \( C_0 \) is not very significant, and conversely, for source releases well inside the canopy. Not surprisingly, the variable \( C_0 \) resulted in concentration values that are lower well inside the canopy and higher above the canopy.

![Fig. 9. LDM calculations for constant and variable \( C_0 \) within the canopy (shown in Fig. 8) for source releases near the canopy top \( (z/h = 0.8, \) top panels) and canopy bottom \( (z/h = 0.2, \) bottom panels) at three distances from the source \( (x/h = 1, 2, 10—left to right) \). Note the large differences in LDM computed mean concentration profiles when the source is located deep inside the canopy but less so when the source is released near the top.](image-url)
when the source was well inside the canopy. The implications of these calculations for optimizing the value of $C_0$ based on matching LDM calculations to measurements are self-evident and may lead to $C_0$ values that are dependent on source location and dispersion distances.

6. Conclusions

From the analysis here, it is clear that $C_0$ inside dense canopies is reduced primarily due to wake production, followed by finite Reynolds number effects. The short-circuiting of the energy cascade tends to increase $C_0$, though not enough to compensate for the reductions. The fact that the largest reduction in $C_0$ is due to wake production is a mixed blessing. On the one hand, closed-form solutions to the spectral budgets are notoriously difficult given the inverse cascade and energy injection. On the other hand, it is encouraging because the wake production varies with $U$, the least difficult variable to model inside dense canopies.

Simplified scaling arguments were proposed for each of these three effects and tested using flume experiments. The fact that $C_0$ may vary nonlinearly inside canopies complicates inverse estimates of $C_0$ by fitting LDM models to concentration data. These optimized $C_0$ can be sensitive to the source location (especially inside the canopy) and concentration sampling points. On a positive note, the fact that $C_0$ may vary within the canopy does not require any revisions to the well-mixed condition because LDM are not sensitive to gradients in $C_0$.

The issue of anisotropy in $C_0$, however, was not addressed here though it may be pertinent to LDM computations for the CSL. Recent tracer trajectory experiments reported significant anisotropy in $C_0$ for the three directions, with vertical directions appearing to yield lower estimates of $C_0^*$ of 4.8 when compared to longitudinal and lateral ($C_0^* \approx 6.2$) directions (Ouellette et al., 2006b). These findings are somewhat opposite to what was reported by other studies for shear and convectively driven atmospheric flows (Anfossi et al., 2006; Degrazia and Anfossi, 1998) as shown in Table 1. Collectively, these two studies are perhaps suggestive that large-scale anisotropy can still persist at smaller scales and manifests itself in anisotropic $C_0^*$ despite the onset of a Kolmogorov (Lagrangian) scaling. Similar issues have already been pointed out in Eulerian structure function scaling (Katul et al., 1997a; Shen and Warhaft, 2000; Warhaft, 2000). Moreover, these recent Lagrangian measurements are also suggestive that the anisotropy in $C_0^*$ weakly decays with increasing $R_s$ (Ouellette et al., 2006b), which can be problematic for CSL flows.

There are a number of other ‘thorny’ issues not addressed here including (1) the convergence of Eulerian and Lagrangian velocity statistics at finite Reynolds number such as the case inside canopies, (2) whether the relationship between Eulerian and Lagrangian integral time scale only dependent on turbulent intensity inside canopies when spectral short-circuiting and wake production cannot be ignored, and (3) whether the effect of wake production on $C_0$ persists with canopies possessing multiplicity of length scale (vis-à-vis a single rod diameter), and (4) how well such findings hold when the flow statistics vary in 3-D rather than 1-D (vertical in this case). Progress on all these issues can only be achieved with some coordinated effort using numerical simulations and innovative laboratory and field experiments. Hence, the findings here are intended to provide a starting point for future analysis, not ‘finality’ on this topic.

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Appendix. Comparison between two independent methods to estimate the mean turbulent kinetic energy dissipation rate

The dissipation rate profile inside and above the canopy was estimated from the isotropic relationship, given by

$$\varepsilon_m = 15\nu \frac{1}{U^2} \left( \frac{\partial u}{\partial t} \right)^2$$

and was compared to the dissipation rate estimated from the Eulerian energy spectrum given by

$$E_\varepsilon(n) = \bar{\nu}^{2/3} \frac{n}{u_s (\varepsilon_{spec})^{2/3} n^{-5/3}},$$
for $z_a = 0.55$ for $z/h > 1.2$. The intent of this comparison is to independently check the quality of $e_m$ estimates. The $z/h > 1.2$ was chosen here because the spectra in those layers exhibit a clear ISR scaling thereby permitting independent checks on the $e_m$ estimates. The agreement between $\epsilon_{\text{spec}}$ and $e_m$ was encouraging (Fig. A.1, regression equation is $\epsilon_{\text{spec}} = 1.01 e_{\text{iso}} = 2.53 \times 10^{-5}$, with a coefficient of determination $r^2 = 0.98$). Inside the canopy, the $E_R(n)$ no longer scales with $n^{-5/3}$ due to short-circuiting and wake production thereby preventing such comparisons. However, in a separate study but for the same flume configuration, $u_w = 0.037 \text{ m s}^{-1}$, $h_w = 0.6 \text{ m}$, canopy density and height, $e$ was estimated as a residual of the TKE budget and was found to agree well with $e_m$ inside and above the canopy (Poggi et al., 2004a, 2006).

References


