An experimental investigation of the mean momentum budget inside dense canopies on narrow gentle hilly terrain

Davide Poggi a,*, Gabriel G. Katul b

a Dipartimento di Idraulica, Trasporti ed Infrastrutture Civili, Politecnico di Torino, Torino, Italy
b Nicholas School of the Environment and Earth Sciences, Duke University, Durham, NC, USA

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Abstract

Recent theories and model calculations for flows inside canopies on gentle hilly terrain suggest that the impact of advection and pressure perturbations on the mean momentum budget remains problematic when the canopy adjustment length (Lc) is comparable to the hill half-length (L) (referred to as narrow gentle hills). To progress on this problem, detailed laser Doppler anemometry (LDA) and water surface profile measurements were conducted in a large flume simulating a neutrally stratified boundary layer flow over a train of gentle hills covered by a dense canopy with Lc/L ≃ 1. The canopy was composed of an array of vertical cylinders with a frontal area index concentrated in the upper third to resemble a tall hardwood forest at maximum leaf area index. The data was presented in terms of component balance of the mean momentum equation decomposed into a background state and a perturbed state induced by topographic variation. We found that the measured and modelled pressure computed from the topographic shape function were not in phase, with the minimum pressure shifted downwind from the hill summit. We also showed that the recirculation region, predicted to occur on the lee side of the hill close to the ground, was sufficiently large to modify the mean streamlines both within the canopy sub-layer and just above the canopy. This adjustment in mean streamlines can be accounted for through an effective ground concept thereby retaining the usability of linear theory to model the mean pressure gradients. The LDA data suggested that the shear stress gradient remained significant at the bottom of the hill in the deeper layers of the canopy and was the leading term balancing the adverse pressure gradient in the recirculation region. The drag force was the leading contributor to the mean momentum balance near the canopy top and within the deeper layers of the canopy at the hill summit. However, we found that the drag force was not the primary term balancing the adverse pressure gradient within the recirculation zone. Advection was not only substantial above the canopy but remained significant in the deeper layers of the canopy near the hill summit as predicted by recent numerical simulations. In short, no one term in the mean momentum balance can be a priori neglected at all positions across a gentle narrow hill.

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1. Introduction

Air flow inside canopies on complex terrain is a research problem now gaining significant attention in weather predictions (Milton and Wilson, 1996; Belcher and Hunt, 1998; Wood, 2000), ecosystem water and carbon cycling (Raupach, 1992; Finnigan et al., 1990; Finnigan, 2000; Baldocchi et al., 2001; Finnigan and Belcher, 2004), and wind engineering (Kim et al., 2000; Bitsuamlak et al., 2004). One of the distinguishing features of flows inside canopies on complex terrain is that topographic variations produce large-scale changes...
in the mean pressure field, and quantifying the concomitant velocity adjustments in the presence of a canopy remains difficult. There is clearly some urgency in progressing on solutions to the problem of topographically induced transport inside canopies given the need to conduct and interpret micrometeorological measurements of trace gas fluxes over tall forested canopies—many situated on complex terrain (Grace and Malhi, 2002; Finnigan, 2004; Staebler and Fitzjarrald, 2004; Vosper et al., 2005). Because of the non-linearities in the transport equations, even gentle topographic variations can lead to significant changes in the exchange rates of energy, mass, and momentum between a vegetated land surface and the atmosphere (Raupach and Finnigan, 1997; Katul et al., 2006). Lack of progress on this problem is exasperated by the absence of detailed field and laboratory data on rudimentary flow statistics such as the spatial patterns of mean velocity needed for modelling scalar advection. The lack of field data is undoubtedly related to the efforts required to deploy a large array of tall towers and instruments across complex terrain (Feigenwinter et al., 2004; Aubinet et al., 2005). While limited in number, spatial resolution and scope, few laboratory experiments did document the effects of canopy roughness on mean flow patterns above ridges and rough hills (Ruck and Adams, 1991; Finnigan and Brunet, 1995; Castro and Apsley, 1997; Kim et al., 1997; Neff and Meroney, 1998; Athanassiadou and Castro, 2001; Vosper et al., 2005) though data sets on flows inside canopies on gentle hills remain sparse at best.

On the theoretical side, Finnigan and Belcher (2004), hereafter referred to as FB04, recently proposed an analytical model for canopy flow within shallow forested gentle hills. The model predicts a reduced speedup over the hill and an enhanced separation region on the lee side when compared with a similar hill without canopy. These results have been recently confirmed by experiments conducted by Poggi and Katul (2007) and Poggi and Katul (in press-b) (hereafter, the latter study is referred to as PK06). In their simplification to the mean momentum budget equation, FB04 assumed that the advective terms are always negligible inside the canopy and argued that this assumption is reasonable if (i) the canopy is sufficiently dense \((H_c/L_c \gg 1)\), where \(H_c\) and \(L_c\) are the canopy height and adjustment length scale, respectively), and (ii) the topographic variations remain gentle \((H/L \ll 1)\), where \(H\) and \(L\) are the hill height and half-length, respectively). Moreover, FB04 showed that to constrain the vertical velocity imposed by the flow convergence at the canopy top, the forest needs to be sufficiently shallow compared to the hill wavelength \((L_c/L \ll 1)\). These assumptions have been numerically tested by Ross and Vosper (2005), hereafter referred to as RV05, using first order closure principles. RV05 showed that advection is negligible for shallow forests over gentle hills but, as anticipated by FB04, cannot be neglected for a narrow forested hill. As pointed out by RV05, \(L_c/L \ll 1\) is a highly restrictive assumption in practice and conditions for which \(L_c/L \sim 1\) remain crucial to investigate because these are the conditions that promote advection inside canopies.

Both analytical predictions in FB04 and numerical simulations by RV05 showed the existence of a region close to ground on the lee side of gentle hills of reversed flow when \(H_c/L_c \gg 1\). According to FB04, the increased displacement of the streamlines due to a recirculation zone can also impact the magnitude and the phase of the pressure perturbation. Recently, PK06 showed experimentally that this asymmetry in the flow field pattern leads to a phase shift in the measured pressure perturbation when compared to the terrain surface in the case of a narrow hill.

In essence, when viewed from the mean momentum balance perspective, the two terms that modify canopy turbulence from its equilibrium (or flat-world), advection and pressure perturbations, continue to be problematic when \(L_c/L \approx 1\) even for gentle hills \((H/L \ll 1)\). Motivated by this problem and the need for high resolution bench-mark data sets on flows inside dense canopies on gentle but narrow hilly terrain (selected here as \(H/L = 0.1\) and \(L_c/L = 1\)), the study objectives are two-fold: (1) experimentally quantify the impact of gentle hills on the various terms in the mean flow momentum budget within and just above dense canopies; (2) explore the simplifications and scaling arguments used in models of momentum transfer (e.g. FB04) that can be employed in scalar transport calculations.

To pursue these two objectives, detailed flume experiments were conducted to simulate a neutrally-stratified atmospheric boundary layer (BL) flow over a train of hills covered with a dense plant canopy. The BL was experimentally reproduced in a long water channel and the dense plant canopy was simulated using a regular array of vertical cylinders.

2. Experimental facilities

Much of the experimental setup was described in PK06 and Poggi and Katul (2007); however, an overview is provided here for completeness. The experiments were conducted in a recirculating channel
that is 18 m long, 0.90 m wide (=\(b_w\)), and 1 m deep with sides made of glass to permit optical access (Fig. 1). The steadiness of the recirculating flow rate \(Q_r \approx 120 \text{ l s}^{-1}\) was verified by continuous monitoring.

The topography was constructed from a wavy stainless steel wall composed of four modules, each representing a sinusoidal hill with a shape function given by

\[
\tilde{f}(x) = \frac{H}{2} \cos(kx + \pi),
\]

where \(x\) is the longitudinal distance, \(H = 0.08\) m is the hill height, \(k = \pi/(2L)\) is the hill wavelength with \(L = 0.8\) m (Fig. 1). The model canopy is composed of vertical stainless steel cylinders \((H_c = 0.1\) m and diameter \(d_r = 0.004\) m) arc-welded into the sheet wall at a density of a 1000 rods m\(^{-2}\). The vertical distribution of the rods frontal area is designed to resemble a tall hardwood canopy at maximum leaf area index (LAI) with its foliage concentrated in the top third and almost constant in the bottom two-thirds (see Fig. 1).
The longitudinal and vertical velocity time series were measured above the third hill module using a two-component laser Doppler anemometry (LDA). The LDA measurements were performed at 10 positions to longitudinally cover one hill module at 0.40 m from the lateral wall in the spanwise direction, and along a large number of vertical positions (≈35) displaced along a specified coordinate system that adjusts for topography as defined later. The sampling duration and frequency for each run were 300 s and 2500–3000 Hz, respectively, and were deemed sufficient to ensure convergence of the flow statistics (Poggi et al., 2002, 2003). The velocity measurements were conducted at fully-developed turbulent flow conditions characterized by a bulk Reynolds number $Re_b > 1.3 \times 10^5$ (defined here using $h_w = 60$ cm and $u_t = Q_f/(b_w h_w) = 0.22$ m s$^{-1}$ as BL depth and depth-averaged velocity, respectively). We sampled 45 cm (of the 60 cm water level) in the displaced vertical direction. We concentrated the vertical measurement array close to the ground so as to zoom into the inner layer dynamics. To simplify the acquisition procedure, the inclination of the two velocity components was kept constant and equal to the slope of the surface. However, the measurement path from the surface follows the displaced coordinate system. Numerical post-processing of the acquired data was then carried out to re-adjust the two components of the velocity with the theoretical coordinates thereby permitting direct comparisons between theory and data.

2.1. Coordinate systems

While several coordinate systems are possible (e.g. rectangular cartesian or terrain following), a streamline coordinate system (hereafter referred to as a displaced coordinate system) that adjusts according to hypothetical flow dynamics is preferred. The advantages of a displaced coordinate system is that it reduces to terrain-following near the ground and to rectangular cartesian well above the hill. Hence, it retains advantages of both coordinate systems in the appropriate regions (see FB04). The data post-processing employs this displaced coordinate system. With respect to a rectangular coordinate system ($X$ and $Z$), the displaced coordinates ($x$ and $z$) are given by

$$x = X + \frac{H}{2} \sin(kX) e^{-iz};$$

$$z = Z - \frac{H}{2} \cos(kX) e^{-iz},$$

where $H$, $L$, and $k$ are defined in the experiment (Fig. 1). Note that the within canopy flow is bounded by $0 > z > -H_c$.

2.2. Pressure estimation

The longitudinal pressure variations can be computed from detailed water surface profile measurements assuming hydrostatic conditions. We verified the hydrostatic pressure approximation by drilling twenty small holes (1 mm in diameter) at the bottom of the third hill module and connected a small transparent plastic tube to each hole positioned perpendicular to the direction of the mean flow. We found good agreement between the measured water surface levels in each tube and above the hole. We then measured the water surface using a CCD camera mounted on a trolley moving on rails at a speed of 2 m min$^{-1}$ along the entire length of the flume. High resolution (704×576 pixels, DV-AVI with PAL format) digital movies were acquired at high sampling frequency (25 frames/s) along the four test sections using an interrogation window of about 14 cm in width and 12 cm in height. To calibrate the camera position with respect to the hill surface, four runs were conducted using still water conditions. The displacement between the horizontal plane and the water surface was evaluated by post-processing the frame sequences extracted from the acquired movies. To further analyze the phase relationship between the water surface and the hill surface, a second camera was used to acquire the vertical elevation of the ground simultaneously with the first camera.

3. Analysis of the governing equations

To fingerprint the influence of topographic variations on the flow statistics, the mean longitudinal momentum budget equation is often decomposed into an unperturbed background state and a perturbed state produced by topographic variations. The reason this decomposition is employed in the context of canopy flows is that the background state is highly inhomogeneous in the vertical thereby masking smaller longitudinal variations produced by the hill. This decomposition is a legitimate mathematical approach; however, its usefulness is linked with the selection of the background state. In general, the background state is not unique and a number of possibilities exist when dealing with flows over complex terrain. From a Reynolds decomposition perspective, a background state should be defined as the planar average along a hill wavelength so that the spatial mean of the
hill-induced mean velocity perturbations is identical to zero. This definition lacks any prognostic skill because it a priori requires knowledge of the mean flow field across the entire hill. More common to models of isolated hills is a background state referenced to the undisturbed upstream conditions. While this definition is unambiguous in the case of an isolated hill, it remains ambiguous in complex terrain (Ayotte, 1997). In fact, for a train of hills, the upstream velocity profile colliding with the nth hill is disturbed by the previous nth – 1 hill. In short, both definitions are plausible and offer several advantages and disadvantages—with one definition being primarily diagnostic while the other being prognostic. Because of this prognostic potential, we adopt the definition of a background state that offers several advantages and disadvantages—with one definition being primarily diagnostic while the other being prognostic. The background mean momentum equation defines the background area given by (Thom, 1971; Massman, 1997)

\[ U_b(z) = \frac{u^*}{k_v} \ln \left( \frac{z + d}{z_0} \right), \]  

where \( d \) and \( z_0 \) are the zero-plane displacement and momentum roughness heights, respectively. The \( U_b(z) \) in a neutrally-stratified BL flow within a canopy over a flat terrain can be estimated by

\[ U_b(z) = U_b e^{(u(z)/(2p^2))}, \]  

where \( U_b \) is the mean velocity at the canopy top, \( \beta = u^*/U_b \) is a parameter representing the ‘momentum flux’ into the canopy, and \( \zeta(u, z) \) is the leaf drag area per unit ground area given by (Thom, 1971; Massman, 1997)

\[ \zeta(u, z) = \int_0^z L_c^{-1}(u, z') \, dz', \]  

where \( L_c = [C_d(u, z)\alpha(z)]^{-1} \) is the canopy adjustment length scale, and \( C_d \) is the drag coefficient that can vary with the local mean velocity and distance from the ground. When \( L_c \) is constant within the canopy, Eq. (7) reduces to the familiar exponential velocity profile and \( \zeta(u, z) \) collapses to \( \zeta(u, z) = C_d \alpha LAI \), where \( LAI \) is the leaf area index (Massman, 1997; Poggi and Katul, in press-b). The parameter \( \beta \) is known to vary with the canopy density reaching an asymptotic value \( \beta \approx 0.3 \) with increasing canopy density.

### 3.1. Background longitudinal velocity

The \( U_b(z) \) above dense canopies is often evaluated using the classical logarithmic shape given by

\[ U_b(z) = \frac{u^*}{k_v} \ln \left( \frac{z + d}{z_0} \right), \]  

where \( d \) and \( z_0 \) are the zero-plane displacement and momentum roughness heights, respectively. The \( U_b(z) \) in a neutrally-stratified BL flow within a canopy over a flat terrain can be estimated by

\[ U_b(z) = U_b e^{(u(z)/(2p^2))}, \]  

where \( U_b \) is the mean velocity at the canopy top, \( \beta = u^*/U_b \) is a parameter representing the ‘momentum flux’ into the canopy, and \( \zeta(u, z) \) is the leaf drag area per unit ground area given by (Thom, 1971; Massman, 1997)

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### 3.2. Canopy drag parametrization

For \( F_{CB}(z) \), the Reynolds number is often assumed sufficiently large inside the canopy so that the viscous drag can be neglected relative to the form drag resulting in

\[ F_{CB}(z) = C_d(u, z)\alpha(z)U_b^2(z). \]  

This formulation for \( F_{CB} \) can account for viscous effects through variations in \( C_d \). Nevertheless, these effects are neglected here.

### 3.3. Background shear stress

Within a dense canopy on flat terrain (i.e. \( \frac{\partial \tau_b}{\partial x} = 0 \)), the turbulent shear stress profile can be written as

\[ \tau_b = U_b^2 \beta^2 e^\zeta/\beta^2. \]
To ‘close’ the parameterizations for the background mean velocity and shear stress, two variables must be specified $-d$ and $z_0$. These two variables are often determined by imposing continuity and smoothness on $U_b$ at the canopy top resulting in $d = 2k_v \beta^3 L_c(0)$ and $z_0 = d e^{-k_v \beta}$.

To summarize, the above formulations for background variables are derived for dense canopies on flat terrain with an $L_c$, $\beta$, $d$, and $z_0$ assumed to be independent of topographic variations when being applied to non-flat terrain conditions except through $u_s$ (Wilson et al., 1998; Finnigan and Belcher, 2004). Hence, before proceeding to the discussion of the hill-induced perturbations, it is imperative to compare $U_b$ and $\tau_b$ obtained from these canonical formulations with their spatially-averaged counterpart.

In Fig. 2a and b, the measured longitudinal velocity and shear stress profiles are shown for all 10 locations across the hill. The background velocity and shear stress profiles determined by spatially averaging the data ($U_{sa}(z)$) and from the exponential profile (Eqs.(6) and (10)) are compared. In evaluating $U_b(z)$ and $\tau_b$, we used the measured $u_s = 0.038$ ms$^{-1}$ (extrapolated using the maximum measured value of $\tau_b(z)$), and assumed $\beta = 0.3$ (Poggi et al., 2004). The good agreement between $U_b(z)$ and $U_{sa}(z)$ suggests that the background velocity retains its canonical logarithmic shape above the canopy and its exponential shape inside the canopy for a train of gentle hills and suggests that $U_b$ and $\tau_b$ remain acceptable descriptors of the background state here.

### 3.3.1. The perturbed pressure field

As earlier stated, one of the distinguishing features of flows inside canopies on complex terrain is that topographic variations produce pressure perturbations thereby making $\Delta p$ the primary forcing in the perturbed mean momentum budget equation. The simplest approximation is to assume that $\Delta p$ is strictly ‘external’ derivable from topographic variations and independent of canopy and flow configuration (e.g. FB04). There is now evidence from closure model calculations (RV05) and flume experiments (PK06) that an asymmetry exists between the perturbed pressure and topography that is intimately linked with the recirculation region at the lee side of the hill. However, in a first order analysis, $\Delta p$ remains primarily driven by terrain variation and is often modelled through a characteristic magnitude, $p_0$, and a dimensionless longitudinal function representing the leading-order topographic variations, $\sigma(x)$, given by

$$\Delta p(x) = p_0 \sigma(x).$$

In Eq. (11), $p_0 = U_0^2$, where $U_0$ is a characteristic velocity representing the forcing due to the mean flow field in the outer region. Note that the spatial variation of the pressure only depends on the longitudinal coordinates because the inner layer (formally defined later) is usually thin compared to $L$. This simplification was confirmed using a numerical model by RV05. For a two-dimensional generic hill, $\sigma(x)$ can be computed in the Fourier domain once the so-called hill shape function $f(x)$ is known using (Jackson and Hunt, 1975;...
Belcher and Hunt, 1998)

\[ \delta(s) = -k \hat{f}(s), \]  

where \( \hat{f}(s) \) is the Fourier transform of \( f(x) \), denoted hereafter by the tilde and is given by

\[ \hat{f}(s) = \int_{-\infty}^{\infty} f(x) e^{-ixs} \, dx. \]  

Therefore, in real space and for a sinusoidal hill shape function, the pressure becomes,

\[ \Delta p(x) = p_0 \sigma(x) = -U_0^2 \frac{H}{2} k \cos(kx). \]

The pressure perturbation for hills with a general shape function can be evaluated using linear superposition (to a leading order). In particular, once the “ith” most representative modes of a generic hill shape are evaluated, the pressure perturbation becomes

\[ \Delta p_i(x) = -U_0^2 \sum_{n=0}^{\infty} k_n \left( \frac{H_n}{2} \right) \cos(k_n x), \]

where \( H_n/2 \) and \( k_n \) are, respectively, the amplitude and the wave number of the ith mode.

This linear treatment is based on the assumption that the perturbed pressure is a priori imposed by topographic variations. It has been shown by PK06 and RV05 that this assumption is questionable when modelling pressure perturbations for dense canopy flows. In fact, the asymmetry in the flow pattern inside a dense canopy leads to a phase shift in the measured pressure perturbation when compared with its predicted behavior from topography only for a narrow hill. Both PK06 and RV05 also showed that the minimum in the pressure is shifted downhill from the summit in disagreement with predictions by the above linear analysis.

PK06 argued that it is possible to create a reasonable feedback between the asymmetric flow pattern in the sub-canopy region and the outer layer without altering the linear analysis for the pressure perturbations. An effective surface was proposed in which the expected mean flow streamlines close to the ground are used instead of the topographic surface. It was shown that linear theory, when combined with such an effective surface, was able to reproduce the measured pressure variations across the hill. To illustrate, PK06 used three energetic modes in Fourier space to describe this

![Fig. 3](image-url)
effective surface. The effective surface was constructed to follow the topography upwind and adjust to account for the upper recirculation zone area on the lee-side. The dashed line in Fig. 3 represents the superposition of these three modes in real space, taken from PK06. Note the good agreement between predicted and measured pressure variations in Fig. 3 when the pressure is computed from the effective surface. While Fig. 3 shows that measured and predicted pressures agree, the pressure data is not sufficiently accurate to resolve pressure gradients needed in the mean momentum balance. Hence, we depart from the analysis by PK06 and conduct a formal sensitivity analysis on the resultant pressure by varying the relative amplitudes and phases of the two most energetic modes used to describe the effective ground (shaded area) but keep the wavelength unaltered. We found that the scatter in the measured pressure shown in Fig. 3 can be entirely bounded by varying the amplitudes and phase shifts by no more than 10%. We will use this sensitivity analysis in the description of the effective ground to assess how the mean momentum balance closure, as derived from the data, is impacted by uncertainties in the pressure gradient description.

3.3.2. Scaling arguments and dynamical regions

Because of the multiple length and time scales involved in flow over vegetated hills, a natural question arises as to whether all these scales play a dominant role in the flow dynamics or whether some scaling arguments can be used to further simplify the momentum budget equation. For flow over bare hills, a simplification can be achieved by decomposing the boundary layer over hills into several distinct regions—each representing a balance between the leading terms in the mean longitudinal momentum equation. Such analysis was successfully used over the last three decades (Jackson and Hunt, 1975, hereafter referred to as JH75) to derive analytical solutions for the mean flow (Hunt et al., 1988; Belcher and Hunt, 1993, for a complete discussion). This approach also forms the basis for numerous simplifications in FB04 and is briefly reviewed next.

Linear analysis in JH75 for flows on hilly terrain leads to two distinct regions known as outer and inner regions. These two regions emerge from time scale arguments associated with the relative adjustment of the mean and turbulent flow to topographic perturbations. In particular, Belcher and Hunt (1993) introduced the mean distortion (TD) and the Lagrangian integral (TL) time scales to explore how the mean flow and turbulence adjust to topographic variations within these two regions. TD characterizes the distortion of turbulent eddies by the straining motion associated with spatial variability in the mean flow induced by the hill. It represents the characteristic time that the mean flow field needs to stretch large eddies through work done by advection against the mean spatial velocity gradients. TL characterizes the time scale of classical turbulent stretching (or relaxation) of large eddies due to the action of a local mean-flow velocity gradient. Stated differently, TL is the time that turbulent fluctuations need to come to equilibrium with the local mean velocity gradient (Tennekes and Lumley, 1972, Chapter 3). The ratio TD/TL is traditionally used to separate outer from inner regions, as discussed next.

3.3.2.1. Outer region. In the region where TD/TL \( \gg 1 \), local stretching of large eddies is much slower than the distortion due to advection. This region is called the rapid-distortion region or the outer region and is characterized by flow dynamics governed by the balance between advection and the pressure gradient terms, with turbulent stresses playing a minor role. The turbulent flow is rapidly distorted and a direct proportionality between the hill shape and the flow statistics can be assumed using \( \Delta \tau / \bar{u}^2 \sim \Delta u / U_b(L) = O(H/L) \) (Britter et al., 1981). For gentle hills, \( H/L \ll 1 \) and the relative importance of turbulent stress gradients and longitudinal advection on the mean longitudinal momentum balance can be approximated as

\[
\frac{U_b(z) \partial \Delta u / \partial x}{\partial \Delta \tau / \partial z} = \left( \frac{U_b(L)}{u_*} \right)^2 \gg 1,
\]

thereby leading to a simplified mean momentum equation:

\[
U_b(z) \frac{\partial \Delta u}{\partial x} + \Delta w \left( \frac{\partial U_b(z)}{\partial z} \right) = -\frac{\partial \Delta p}{\partial x},
\] (17)

3.3.2.2. Inner region. When TD/TL \( \ll 1 \), the local stretching of large eddies is fast enough to compete with the distortion due to the mean flow advection. This region is called the local-equilibrium region or inner region because the local eddies relax to equilibrium with the local mean velocity gradient before spatial advection can transport and stretch them. Because of this equilibrium, K-theory can be used to predict perturbations in the turbulent stresses. Spatially, the inner region is defined for \( z < h_i \), where \( h_i \) is known as
the inner layer depth estimated by solving
\[ h_i \simeq \frac{2k_v^2}{L \ln(h_i/z_0)}. \] (18)

Within the inner region, Belcher and Hunt (1998) evaluated the ratio of the perturbation stress gradient (using first order closure principles) to longitudinal advection to give
\[ \frac{U_b(z) \partial u / \partial x}{\partial \Delta \tau / \partial z} = \frac{U_b(h_i) \Delta u / L}{(2k_v u_i h_i \Delta u / L) / h_i} = \frac{U_b h_i}{2k_v u_i} > 1. \] (19)

Based on this analysis, the perturbation in the stress gradient still plays a minor role in the momentum budget equation when compared with longitudinal advection, but it is not negligible anymore. Hence, in the inner region, the mean flow is governed by both advection and Reynolds stresses.

3.3.2.3. Canopy sub-region. The canopy sub-layer complicates the inner layer scaling by introducing a new term in the mean momentum budget equation \( F_C \) and by altering the lower boundary condition (i.e. the canopy is porous and does not respect the no-slip condition). This new term clearly imposes new characteristic length scales such as \( H_c \) and \( L_c \). A logical question then is whether the addition of these new length scales still permit simplifications to the mean momentum budget equation.

According to FB04, the advective terms can be neglected in the canopy sub-region based on the following length scale argument: vertical variation of the turbulent shear stress scales with the canopy mixing length, \( l = 2b^3 L_c \) while longitudinal variations (mainly advection) continues to scale with \( L \). Therefore, the ratio of the mean-flow advection to perturbed stress gradient can be estimated as
\[ \frac{U_b(z) \partial u / \partial x}{\partial \Delta \tau / \partial z} \simeq \frac{U_h \Delta u / L}{(2b^2 U_h \Delta u) / l} = \frac{l}{2b^2 L} = \frac{L_c}{L} \ll 1, \] (20)

where classical gradient-diffusion closure assumptions are employed for the stress perturbation and \( U_h \) is estimated from the background velocity at the canopy top \( (U_h = U_b(0)) \). The restriction \( L_c/L \ll 1 \) suggests that advection may be negligible when the canopy drag...
length scale is very small compared to the hill width. As noted by RV05, this condition is highly restrictive in practice and calls into question the arguments for neglecting advection inside canopies. RV05 showed that for a gentle but narrow vegetated hill ($H/L = 0.02$ and $L_c/L > 1$) the streamlines of the flow field do not follow the geometry of the canopy top. Contrarily, the pattern of the flow field shows a steep horizontal convergence just before and a sharp divergence just after the summit of the hill. This phenomenon, which occurs at length scales much shorter than $L$, is associated with the asymmetry in the pressure arising from the recirculation region (see also data in PK06).

The negligible advection argument proposed by FB04 hinges on the assumption that within the canopy sub-region, distortion of typical vortices due to advection occurs at time scales much longer than the stretching due to the local turbulence (i.e. $TD \gg TL$). For narrow hills, the results in RV05 suggest that some regions within the canopy experience sudden variations in turbulence regimes (e.g. the hill top) that can occur on short time scales. The consequence of this sudden variation is that the distortion time and length scales can be locally much smaller than expected. Hence, using the length scale arguments in Eq. (20), advection may be significant if longitudinal variations in $\Delta u$ occur on length scales shorter than $L$. Here, we will use the data to directly explore the relative importance of advection with respect to the other terms in the mean momentum balance when $L_cL^{-1} \approx 1$.

4. Investigating the mean momentum balance

From the LDA velocity measurements, we computed the sum of the two advective terms (i.e. the terms on the left hand side in Eq. (8)) and the drag force assuming $C_d = 0.3$ as in Poggi et al. (2004), and from the effective ground in Fig. 3, we computed the mean pressure gradient. Because of inherent uncertainties in estimating the mean pressure gradient, we used variations in the effective ground (also shown in Fig. 3) to compute plausible variations in the pressure gradient as part of a sensitivity analysis on the momentum closure. The shear stress gradients were then computed from the mean momentum balance (i.e. Eq. (5)) as a residual (shown Fig. 4) and hereafter denoted by $d\tau_{eff}/dz$. To
assess the degree of closure in the measured mean momentum balance, Fig. 5 compares the shear stress derived from the LDA measurements ($\tau_m$) with the shear stress derived by vertically integrating $d\tau_{\text{eff}}/dz$ in Fig. 4 assuming zero-stress at the ground. Fig. 5 also shows the shear stress ($\tau_{\text{grd}}$) profile but assuming that the pressure follows the topographic hill shape function. As expected, the agreement between $\tau_{\text{eff}}$ and $\tau_m$ is by far superior than the agreement between $\tau_{\text{grd}}$ and $\tau_m$. Furthermore, the variations in the pressure shown in Fig. 3, originating from uncertainties in the effective ground, do lead to variations in $\tau_{\text{eff}}$ that entirely bound $\tau_m$ at all positions across the hill. Hence, Fig. 5 demonstrates that once the pressure is well parameterized, along with an adequate $C_d$ and the measured leaf area index, acceptable mean momentum balance closure can be achieved from the data.

Fig. 4 permits us to explore the relative importance of the various terms in the mean momentum equation across the hill within and above the canopy. From Fig. 4, it is safe to say that no one term can be neglected at the outset at all positions across the hill. However, the relative importance among the various terms clearly varies in space and not all terms are significant at all positions. Based on the measurements in Fig. 4, we found that:

1. The shear stress gradient remains among the leading terms at the bottom of the hill in the deeper layers of the canopy. Furthermore, it appears to be a significant term balancing the adverse pressure gradient in the recirculation region.
2. The drag force is among the leading contributors near the canopy top and within the deeper layers of the canopy at the hill summit. However, it is not the major term balancing the adverse pressure gradient within the recirculation zone.
3. Advection is not only substantial above the canopy but can be significant in the deeper layers of the canopy near the hill summit as predicted by RV05 for narrow hills.

While the uncertainty in the computed shear stress gradients in Fig. 5 appears large (i.e. sizable shaded area) when compared to the LDA measurements, its impact on the above three findings is not significant.

![Fig. 6. Top panel: The streamlines computed from the LDA measured mean velocity across the hill illustrating the convergence–divergence zone near the hill summit in the deeper canopy layers, and the recirculation zone. Bottom panel: The measured mean longitudinal velocity and the delineated recirculation zone (white solid line), effective ground (white dot-dashed line), canopy sub-layer (black dot-dashed line), and the inner layer ($h_i$) depth (black broken line) are also repeated for reference. Longitudinal distances are normalized by the hill half-length ($L$) and vertical distances are normalized by $h_i$.](image-url)
because its origins trace back to uncertainties in the measured pressure gradient.

Given that advection can be significant inside the canopy, we analyzed its 'genesis' using the streamlines computed from the mean velocity measurements. The mean streamlines, shown in Fig. 6, suggest a convergent–divergent zone near the hill summit in the deeper canopy layers. When this zone is interpreted in light of the high velocity (in Fig. 3), it becomes clear why advection is important at the summit. In contrast, while the streamlines rapidly change upstream of the recirculation zone on the lee side, these changes appear ‘dampened’ by the small velocities thereby resulting in negligible advection (despite the large gradients).

5. Summary and conclusions

The problem of air flow inside canopies on complex terrain is presently in a phase of rapid development in terms of analytical theories and numerical model results. This progress bares striking resemblance to earlier progress on flows above isolated gentle hills. The linear theory of JH75 provided a predictive framework of how to proceed in designing field and laboratory experiments. The analytical theory proposed by FB04 and the numerical model results by RV05 for the canopy and inner layers to some extend mirror the role of JH75. Both FB04 and RV05 note that the two terms that modify canopy turbulence from its undisturbed state—advection and pressure perturbations represent a serious knowledge gap when \( L_c/L \approx 1 \) even for gentle hills (\( H/L \ll 1 \)). RV05 labelled these conditions \( (L_c/L \approx 1 \) and \( H/L \ll 1 \)) as gentle narrow hills.

Here, we conducted detailed flume experiments for flows inside canopies on gentle narrow hills to begin exploring the role of pressure gradients (and their concomitant uncertainty) and advection on the velocity perturbations. A logical starting point in this exploration is the mean momentum balance. The data were presented in terms of a component balance of the mean momentum equation decomposed into a background state and a perturbed state induced by topographic variation. We found that the measured and modelled pressure computed from the topographic shape function are not in phase, with the minimum pressure shifted downwind from the hill summit in agreement with model calculations of RV05. We also showed that the recirculation region, predicted by FB04 to occur on the lee side of the hill for the deep canopy layers, modifies the streamlines close to the ground. This adjustment in streamlines can be accounted for through an effective ground thereby retaining the usability of linear theory in pressure calculations. The experiments here strongly supports the use of an effective ground approach for pressure calculations as discussed in PK06. We also showed that the background state can be readily predicted from standard canopy turbulence theories even when the terrain is not an isolated hill. For the train of four hills here, we found that the background velocity predicted from classical canopy turbulence theories agree with the spatially-averaged velocity along one hill wavelength. This finding does have important consequences to simplifying assumptions such as linearizing the advective and drag force terms (as proposed by FB04) currently being explored. The shear stress gradient remains significant at the bottom of the hill in the deeper layers of the canopy and is the leading term balancing the adverse pressure gradient in the recirculation region. On the other hand, the drag force is the leading contributor to the mean momentum balance near the canopy top and within the deeper layers of the canopy at the hill summit. However, it does not play a significant role in balancing the adverse pressure gradient within the recirculation zone. Advection is not only substantial above the canopy (as predicted by FB04 and JH75 for the inner layer) but is significant in the deeper layers of the canopy near the hill summit as predicted by the numerical results in RV05. In short, what the experiments here suggest is that no one term in the mean momentum balance can be neglected at the outset at all positions across the hill inside the canopy.

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