Lecture – 15: Infiltration

Definition:

Infiltration is the process of water entry into the soil system, generally by downward flow through all or part of the soil surface. This process is particularly significant given the role of soil water in sustaining vegetation, agriculture and food production, ground water recharge, etc…

Furthermore, infiltration dictates the antecedent soil moisture, which then regulates surface runoff, flash floods, etc….

Infiltration rate – volume of water that enters the soil per unit ground area per unit time. The infiltration rate (i) is in water depth per time [e.g. mm/hour].

The cumulative infiltration (in mm of water) over a time period T is $I = \int_0^T i(t)dt$.

Infiltration rate can be either supply or soil controlled. That is, if we sprinkle a small amount of water on the soil, it is likely that all the water enters the soil system – and hence, the infiltration process is governed by the amount of water we apply. On the other hand, during flooding events, the precipitation rate is so high and it exceeds the ability of the soil to transmit this water. For such cases, the infiltration rate is "soil limited".

Infiltration capacity is measured by the equality between the supply rate and soil controlled infiltration rate. Hence, infiltration capacity is often calculated to assess whether the infiltration is supplied controlled or soil controlled.

Models of Infiltration Capacity:

- *Green and Ampt (1911):*

Earliest equation was proposed by Green and Ampt (1911) in which the infiltration capacity is:

$$i = i_c + \frac{b}{I}$$

where i and I are the infiltration rate and cumulative infiltration, respectively, i_c is analogous to the saturated hydraulic conductivity (note: at large times, $I \to \infty$,and $i = i_c$), and b is an empirical parameter. Note, time does not explicitly appear in the Green-Ampt equation. The model can be re-formulated so that:

$\frac{dI}{dt} = i_c + \frac{b}{I}$, and hence time becomes explicit. Naturally, by re-arranging the equation

$\frac{dI}{i_c + \frac{b}{I}} = dt$, and integrating both sides:

$$t = \frac{I}{i_c} - \frac{b}{i_c^2}\log(i_c I + b)$$

- Kostiakov (1932):

The next equation was proposed by Kostiakov, and is given by:

$$i = Bt^{-n}$$

where B and n are constants. The Kostiakov model is flawed at the two end-members:
$t \to \infty$, $i \to 0$,
$t \to 0$, $i \to \infty$.

The cumulative infiltration is given as:

$$I = \frac{B}{1-n}t^{1-n}$$

- Horton (1940):

The Horton model is a 3 parameter model derived to alleviate the problems in the asymptotic limits of the Kostiakov model.

$$i = i_c + (i_o - i_c)e^{-\beta t}$$

i_c = saturated hydraulic conductivity
i_o = initial infiltration rate (or i at t=0)
β = constant that varies with soil type and soil cover.

- Philip (1957): Using an analytical solution to Richard's equation for soil moisture redistribution: $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K(\theta)\frac{\partial H}{\partial x}\right)$, where θ is the volumetric soil moisture, K is the hydraulic conductivity function, and H is the total energy, Philip (1957) demonstrated that:

$$i = K_s + \frac{1}{2}S_p t^{-1/2}$$

where i is the infiltration capacity, K_s is the saturated hydraulic conductivity, and S_p is the sorption coefficient.

Note, for short times, $t^{-1/2}$ is very large and sorption controls the infiltration capacity (i.e. $i \approx \frac{1}{2}\frac{S_p}{\sqrt{t}}$). For large times, $i \to K_{sat}$.

The cumulative infiltration capacity is given by:

$$I = K_s t + S_p t^{1/2}$$

- SCS Model:

The Soil Conservation Service (SCS) model is a revision of the Philip model, and is given by

$$i = a t^b + c$$

where a, b, and c are empirical constants. Note – for $b = -1/2$, one recovers the Philip model.