Lecture – 3: Areal Estimation of Precipitation

1. Introduction:

One important aspect of hydrologic modeling is the estimation of the total precipitation and its distribution within a watershed. This problem is commonly referred to as “areal estimation of precipitation” and is best described as follows:

*Given the coordinates of m precipitation stations along with their respective recorded precipitation values \( P_i; \ i = 1, \ldots, m \), how can we determine the area-averaged \( P_{\text{avg}} \) precipitation from these limited stations?*

This problem is illustrated below, where \( P_{\text{avg}} \) across the entire basin has to be estimated from measured \( P_1, P_2, P_3, \) and \( P_4 \).

We will discuss four methods:

1) Arithmetic Average, 2) Theissen Polygons, 3) Isohyetal Method, and 4) Grid Method.
2. Arithmetic Average:

The simplest approach is to assume that

\[ P_{\text{avg}} = \frac{\sum_{i=1}^{m} P_i}{m} = \frac{P_1 + P_2 + P_3 + P_4}{4}; \]

This method will give reasonable results if the variability among \( P_i \) is not too large. A “rule of thumb” is that if the standard deviation of \( P_i \) is less than 10% of \( P_{\text{avg}} \), then the arithmetic average will be an accurate estimator of \( P_{\text{avg}} \).

The standard deviation (\( \sigma_p \)) is given by

\[ \sigma_p = \sqrt{\frac{\sum_{i=1}^{m} (P_i - P_{\text{avg}})^2}{m}} \]

where \( P_{\text{avg}} \) is given by the arithmetic average.

Hence, the arithmetic average will be accurate when \( \frac{\sigma_p}{P_{\text{avg}}} < 0.1 \).

3. Theissen Polygons:

The Theissen polygon method assumes that each precipitation gage does not get the same weight as in the arithmetic method.

\[ P_{\text{avg}} = \sum_{i=1}^{m} w_i P_i \]

where \( \sum_{i=1}^{m} w_i = 1 \). The Theissen polygon method REDUCES to the arithmetic method if

\[ w_i = \frac{1}{m} \]

In the Theissen polygon method,

\[ w_i = \frac{A_i}{A_r} \]; where
\[ A_T = \text{Total basin or watershed area.} \]
\[ A_i = \text{Area defined by the Theissen polygons.} \]

To determine \( A_i \), use the following procedure:

1. Join adjacent station locations with straight lines
2. Take the PERPENDUCALR BISECTORS to those lines.
3. Define the polygons bounding each station and compute its area.

If we apply this algorithm to each of the example shown in the INTRODUCTION section, we obtain the above figure. The areas \( A_1 \) … \( A_4 \) are shaded. The total area is 
\[ A_T = A_1 + A_2 + A_3 + A_4 . \]

How to compute the polygon areas becomes a major challenge in this method.

4. Isohyetal Method

The basic formulation is also
\[ P_{\text{avg}} = \sum_{i=1}^{m} w_i P_i \text{, where } \sum_{i=1}^{m} w_i = 1 \]

but the weights are defined by the contour map area as shown below and \( P_i \) is the representative contour.
where the 300, 400, …. are contour lines of precipitation.