How well do stomatal conductance models perform on closing plant carbon budgets? A test using seedlings grown under current and elevated air temperatures

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Future carbon and water fluxes within terrestrial ecosystems will be determined by how stomatal conductance (gs) responds to rising atmospheric CO2 and air temperatures. While both short- and long-term CO2 effects on gs have been repeatedly studied, there are few studies on how gs acclimates to higher air temperatures. Six gs models were parameterized using leaf gas exchange data from black spruce (Picea mariana) seedlings grown from seed at ambient (22/16°C day/night) or elevated (30/24°C) air temperatures. Model performance was independently assessed by how well carbon gain from each model reproduced estimated carbon costs to close the seedlings’ seasonal carbon budgets, a ‘long-term’ indicator of success. A model holding a constant intercellular to ambient CO2 ratio and the Ball-Berry model (based on stomatal responses to relative humidity) could not close the carbon balance for either treatment, while the Jarvis-Oren model (based on stomatal responses to vapor pressure deficit, D) and a model assuming a constant gs each closed the carbon balance for one treatment. Two models, both based on gs responses to D, performed best overall, estimating carbon uptake within 10% of carbon costs for both treatments: the Leuning model and a linear optimization model that maximizes carbon gain per unit water loss. Since gs responses in the optimization model are not a priori assumed, this approach can be used in modeling land-atmosphere exchange of CO2 and water in future climates.


1. Introduction

Descriptions of stomatal responses to environmental drivers are needed when estimating the simultaneous exchange rates of heat, carbon dioxide, and water vapor between terrestrial ecosystems and the atmosphere [Sellers et al., 1995, 1996; Baldocchi and Meyers, 1998; Lai et al., 2000; Siqueira and Katul, 2002; Juang et al., 2008]. Studies showing that rising atmospheric CO2 reduces, or does not affect, leaf-scale stomatal conductance (gs) are numerous, spanning over 100 years [Darwin, 1898; Scarth, 1927; Meidner, 1987; Ellsworth et al., 1995; Heath, 1998; Medlyn et al., 2001; Schäfer et al., 2002; Wullscheleger et al., 2002; Ainsworth and Rogers, 2007; Konrad et al., 2008]. More importantly, the implications of reductions in gs with rising atmospheric CO2 for global carbon and water cycles have been considered within the context of global climate models, suggesting increased continental scale runoff and a positive feedback on rising air temperatures [Cox et al., 2000; Gedney et al., 2006; Betts et al., 2007].

However, stomatal responses to another concomitant global change factor, rising air temperatures, are less certain. With increasing leaf temperatures, gs can increase, decrease, show a peaked function, or remain relatively constant [Kemp and Williams, 1980; Monson et al., 1982; Sage and Sharkey, 1987; Santrucek and Sage, 1996; Cowling and Sage, 1998; Day, 2000; Yamori et al., 2006; Weston and Bauerle, 2007; Kubien and Sage, 2008; Way and Sage, 2008a, 2008b; Mott and Peak, 2010; Silim et al., 2010], although this response is often complicated by responses of gs to vapor pressure deficit (D) when D is not controlled during measurements. The mechanism driving stomatal sensitivity to short-term changes in leaf temperature is still a matter of debate [Fredeen and Sage, 1999; Peak and Mott, 2011; Pieruschka et al., 2010]. But even if the short-term (minutes to hours) response of gs to a change in leaf temperature was predictable, long-term acclimation to different growth temperatures can alter the response of stomata to leaf temperature. Plants
grown at different thermal regimes often have a similar shape in the response of $g_s$ to leaf temperature, although with differing values of $g_s$ [Kemp and Williams, 1980; Yamori et al., 2006]. However, leaves acclimated to different growth temperatures can show differing, or even opposite, patterns in how $g_s$ responds to increasing leaf temperatures [Santrucek and Sage, 1996; Way and Sage, 2008a; Stitt et al., 2010]. While the precise pathways responsible for these differences in $g_s$ values and $g_s$ response patterns remains to be explored, elevated growth temperatures may alter plant allometry in a predictable way, with impacts on plant water use. Way and Oren [2010] found that elevated growth temperatures led to relative increases in leaf mass and relative reductions in root mass, which if not countered by other hydraulic adjustments could affect stomatal control by reducing the ability of trees to transport sufficient water to their leaves to match evaporative demand.

Stomatal responses to changes in environmental variables, such as air temperature, are often predicted with a number of commonly used models [Damour et al., 2010]. One type of model is a hydro-mechanical model that uses epidermal and whole-plant water relations and leaf biochemistry to estimate $g_s$ [e.g., Buckley et al., 2003]. A second category of models uses semi-empirical formulations to link $g_s$ to environmental parameters [Jarvis, 1976] or linearly to the photosynthetic rate ($A$) [Ball et al., 1987; Collatz et al., 1991; Leuning, 1995]. These semi-empirical models are widely used in current climate, hydrologic, and ecosystem carbon models [Sellers et al., 1995; Baldocchi, 1997; Anderson et al., 2000; Luo et al., 2001; Whitehead et al., 2001; Reichstein et al., 2003; Blanken and Black, 2004; Keenan et al., 2010]. A third category assumes that stomata optimally and autonomously regulate their aperture to maximize carbon gain at a given water loss rate, without explicitly resolving the pathways and biochemical signaling mechanisms responsible for stomatal opening and closure [Givnish and Vermeij, 1976; Cowan, 1978; Cowan and Farquhar, 1977; Hari et al., 1986]. Unlike semi-empirical models, the optimization approach does not a priori assume how $g_s$ responds to environmental drivers from existing sets of data, but attempts to derive such responses based on an optimality hypothesis. This point may be especially important with climate change, as plants develop under conditions outside of their current range of air temperatures and CO$_2$ concentrations.

While some of these stomatal models are widely used, studies evaluating their relative performances are uncommon, especially under future climate conditions. Medlyn et al. [2001] studied how two semi-empirical models performed on trees from ambient and elevated CO$_2$ concentrations, concluding that the sensitivity of $g_s$ to environmental parameters (including $D$ and CO$_2$) was unchanged by growth CO$_2$ in the Jarvis model, as was the relationship between $g_s$ and photosynthesis in the Ball-Berry model. Katul et al. [2010] compared two semi-empirical models (the Ball-Berry and Leuning models) with an optimization approach on Pinus taeda grown at ambient and elevated CO$_2$ concentrations, and found that the optimization approach described the data at least as well as the semi-empirical models, provided the cost-of-water parameter linearly increased with increasing CO$_2$. Last, Nijs et al. [1997] compared semi-empirical models and a water-use-efficiency maximization approach for Lolium perenne grown at ambient conditions, elevated CO$_2$, elevated temperature, or both high CO$_2$ and temperature, and found that the Leuning model performed better than either the Ball-Berry model or the model that maximized instantaneous water use efficiency (WUE = photosynthesis/transpiration, $A/E$). The approach of both Katul et al. [2010] and Nijs et al. [1997] was to focus on instantaneous gas-exchange measurements (on the scale of minutes to an hour), comparing predictions and measurements of $A$, $E$, and intercellular CO$_2$ concentrations. However, evaluating the performance of these models over time-scales commensurate with changes in growth and carbon stocks (weeks to months) remains a challenge because of both endogenous (e.g., acclimation effects and leaf area development) and exogenous (e.g., large changes in environmental variables) effects. This evaluation requires confronting models with an extensive data set of both eco-physiological parameters and plant growth measurements to determine which model best predicts the carbon uptake necessary to close the carbon budget of the plant over a long period (e.g., months or longer). Because this suite of data is uncommon for one growth temperature, let alone for conditions similar to current and future climates, we are unaware of any attempts to compare existing $g_s$ models using this approach.

A data set on the growth and physiological parameters of black spruce (Picea mariana (Mill.) B.S.P.) is used here to compare the performance of six commonly used models for predicting $g_s$ and $A$ under ambient and elevated air temperature conditions. Data were derived from seedlings grown at either temperatures representing the species’ current range or temperatures representing predicted boreal conditions for the year 2100 [Way and Sage, 2008a, 2008b]. The same measured photosynthetic and respiration parameters were used within a growth temperature to explore how well each $g_s$ model predicted carbon uptake in plants grown under either current or warming conditions. Because gas exchange measurements were used to determine physiological parameters for the various models, model performance was judged by closure of the seedling carbon budget. Biomass changes occur on much longer time scales than the diurnal variations in meteorological drivers of $g_s$, making plant growth an appropriate scale to evaluate integrated long-term performance of such models. Unlike mature trees or ecosystems, in situ tracking of changes in biomass and carbon fluxes in seedlings can be quantified with relative ease and accuracy, making them an attractive system for exploring the potential of judging stomatal model performance on seasonal or other long-term timescales.

2. Methods

2.1. Data Description

While much of the experimental setup is described elsewhere [Way and Sage, 2008a, 2008b], the salient features most pertinent to the $g_s$ model calibration and evaluation are reviewed here. To model seasonal carbon gain and costs, data from two experiments where well watered black spruce seedlings were grown at either current or elevated growth temperatures were used [Way and Sage, 2008a, 2008b]. Seedlings were grown in greenhouses and growth chambers under ambient CO$_2$ concentrations ($\sim$380 ppm) at $22^\circ$C days
Table 1. Parameters, Parameter Values, and Temperature Correction Equations Used in Modeling Carbon Fluxes for Each $g_s$ Model, Along With References for Those Values and Corrections$^a$

<table>
<thead>
<tr>
<th>Parameter (units)</th>
<th>AT</th>
<th>HT</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{cmx}}$ ($\mu$mol m$^{-2}$ s$^{-1}$)</td>
<td>Arr 33.0 58520</td>
<td>23.3 58520</td>
<td>Mean $V_{\text{cmx}}$ at 25°C from Way and Sage [2008a, 2008b]; activation energy from von Caemmerer and Quick [2000]; Jordan and Ogren [1981]; activation energies from Jordan and Ogren [1984]</td>
</tr>
<tr>
<td>$K_c$ ($\mu$mol m$^{-1}$)</td>
<td>Arr 419 81655</td>
<td>419 81655</td>
<td></td>
</tr>
<tr>
<td>$K_o$ (mmol mol$^{-1}$)</td>
<td>Arr 381 15632</td>
<td>381 15632</td>
<td>Yamori et al. [2006]</td>
</tr>
<tr>
<td>$\Gamma_*$ ($\mu$mol m$^{-2}$ s$^{-1}$)</td>
<td>poly 0.0021 0.1083 2.5821 9.8365</td>
<td>0.0012 0.0613 1.8469 14.348</td>
<td>Acosta et al. [2008]</td>
</tr>
<tr>
<td>$R_{\text{day}}$ shoot (gC m$^{-2}$ d$^{-1}$)</td>
<td>$Q_{10}$ 0.00454 2.2</td>
<td>0.00501 2.2</td>
<td>R$<em>{\text{day}}$ and assumed $Q</em>{10}$ values, Tjoelker et al. [1999]</td>
</tr>
<tr>
<td>$R_{\text{day}}$ root (gC m$^{-2}$ d$^{-1}$)</td>
<td>$Q_{10}$ 0.0245 2.2</td>
<td>0.0322 2.2</td>
<td></td>
</tr>
</tbody>
</table>

$^a$AT, ambient temperature treatment; HT, high temperature treatment; $T_i$, leaf temperature in °K. Equation types (type): Arrhenius (Arr): $y = ab^{(T - 298)/2.34}$; Polynomial (poly): $y = [a(T - 273)^2] - [b(T - 273)^3] + c(T - 273) + d$; Constant (con): $y = a$; $Q_{10}$ equation ($Q_{10}$): $y = ab^{(T - 273)^2}$.

and either 14°C or 16°C nights (ambient temperature; AT) or 30°C days and either 22°C or 24°C nights (high temperature; HT). The data set for each treatment included about 1200 individual gas exchange measurements of $g_s$ and $A$ made at a range of light, CO$_2$, $D$ and temperature conditions, measured with a portable photosynthesis device (Li-6400 and 6400-05, Li-cor Inc., Lincoln, Nebraska, USA). The data also included measurements of $V_{\text{cmx}}$ (the maximum carboxylation efficiency of Rubisco), and the responses of $V_{\text{cmx}}$ to changes in leaf temperature between 10°C and 40°C for each treatment. Table 1 summarizes the derived temperature response curve of $V_{\text{cmx}}$ for each treatment. Growth trajectories for each treatment were also assessed, with shoot height, stem diameter, specific leaf area, and leaf, stem, and root biomass measured at multiple points over the growing season. Growth data from these two experiments were supplemented with height and biomass data from a set of black spruce seedlings simultaneously grown from seed in the same greenhouses as in the work by Way and Sage [2008a] but potted in fine gravel and frequently fertilized (D. A. Way and R. F. Sage, unpublished data, 2007).

### 2.2. Photosynthesis

[8] The basic leaf photosynthesis equation of the Farquhar et al. [1980] model can be expressed as:

$$A = a_1 \left( \frac{C_i - \Gamma_*}{a_2 + C_i} \right)$$

(1)

where $A$ is the leaf biochemical demand for CO$_2$, $a_1$ and $a_2$ vary depending on whether $A$ is Rubisco- or light-limited, $C_i$ is the intercellular CO$_2$ concentration, and $\Gamma_*$ is the CO$_2$ compensation point in the absence of mitochondrial respiration. When $A$ is light-saturated, $a_1 = V_{\text{cmx}}$ and $a_2 = K_c (1 + O/K_o)$, where $K_c$ and $K_o$ are the Michaelis constants for carboxylation and oxygenation, and $O$ is the ambient oxygen concentration. When $A$ is light-limited, $a_1 = \alpha \phi_{\text{max}} Q$ and $a_2 = 2 \Gamma_*$, where $\alpha$ is the leaf light absorptivity (0.08), $\phi_{\text{max}}$ is the maximum quantum efficiency (0.08), and $Q$ is the photosynthetic photon flux density (or PPFD). These $\alpha$ and $\phi_{\text{max}}$ values are standard taken from Campbell and Norman [1998]. The parameters $\Gamma_*$, $K_c$, and $K_o$ were based on the same kinetics used to model black spruce by Way and Sage [2008b] and Sage et al. [2008] (Table 1) and $O$ was set at 210 mmol mol$^{-1}$.

### 2.3. Gas Transport Between the Atmosphere and Leaves

[8] While equation (1) defines the biochemical demand for CO$_2$, the supply of CO$_2$ molecules transported from the atmosphere into the leaves can be expressed as a Fickian diffusion, given as:

$$A = g_s(C_a - C_i) = g_s C_o (1 - C_i/C_o)$$

(2)

where $C_a$ is the atmospheric CO$_2$ concentration. Here, the leaf boundary layer and mesophyll resistances were neglected when compared to the stomatal resistance (0.3). The realized rate of photosynthesis is determined by the balance between the supply and demand functions (i.e., equations (1) and (2)). However, the two equations are not mathematically closed since $g_s$ must be a priori known to solve for $A$ and $C_i$, necessitating one additional equation. It is this ‘closure’ approximation and how it varies between ambient and elevated air temperature that is most uncertain and frames the compass of this work.

### 2.4. Stomatal Conductance Models and Their Parameterization

[10] By estimating $g_s$ from the various models, their effects on $A$ can be compared and these differences can be projected into seasonal carbon uptake to judge model performance. This approach was used to evaluate differences in predicted photosynthesis and seasonal carbon gain for six $g_s$ (i.e., ‘closure’) models, presented in order of decreasing generality. For each model, the same measured photosynthetic parameters for equation (1) and respiration values for each treatment (Table 1) were used; thus, differences in carbon fixation between models are entirely due to differences in the closure model equation needed for predicting $g_s$, $A$, and $C_i$. 

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The third model was the Ball-Berry model [Ball et al., 1987] where stomata respond to relative humidity (RH) such that:

$$g_s = m_1 \frac{A}{C_a - \Gamma_s} RH + b_1,$$

while the fourth model was the Leuning model [Leuning, 1995], where stomata respond to $D$ (instead of $RH$):

$$g_s = m_2 \frac{A}{C_a - \Gamma_s} \left(1 + \frac{D}{D_o}\right)^{-1} + b_2$$

where $m_1$ and $m_2$ are species (or treatment) specific parameters, $D_o$ is the sensitivity of $g_s$ to $D$, and $b_1$ and $b_2$ are minimum $g_s$ values. We determined $b_1$, $b_2$, $m_1$, and $m_2$ for both treatments by plotting measured $g_s$ against either measured

$$\frac{A}{C_a - \Gamma_s} RH$$

for the Ball-Berry model and calculated the slopes (for $m_1$ and $m_2$) and intercepts (for $b_1$ and $b_2$) using standard least squares regression fitting for data, where PPFD was >400 $\mu$mol m$^{-2}$ s$^{-1}$ and CO$_2$ concentrations ranged from 300 to 500 $\mu$mol mol$^{-1}$, $D/D_o = 0.6$ for the Leuning model [Oren et al., 1999] (Table 2). For each point in the Constant $C_i/C_a$, Ball-Berry and Leuning models, we temperature-corrected $\Gamma_s$ (Table 1) to ensure no bias originated from this quantity, since $\Gamma_s$ is temperature sensitive. Analytical solutions for $A$ for the Ball-Berry and Leuning models can be found in the Appendix.

The fifth model (Jarvis-Oren) was also based on ‘prescribed’ $g_s$ responses to $D$, from AT and HT seedlings measured at near ambient CO$_2$ concentrations (300–500 $\mu$mol mol$^{-1}$) and PPFD values >400 $\mu$mol m$^{-2}$ s$^{-1}$, consistent with the Ball-Berry and Leuning models [Way and Sage, 2008a, 2008b]. The approach of Jarvis [1976], as used by Oren et al. [1999], was employed to represent $g_s = g_{sref} (1 – mlnD)$, where $g_{sref}$ is $g_s$ at a $D$ of 1 kPa (reference $D$), where $m$ was shown to be a near-constant across more than 60 species and varies between 0.5 to 0.6. Because the relationship between $g_s$ and $\ln D$ varied with leaf temperature (Figures 1a and 1b), the data could not be readily described with a single function and the $g_s$ measurements were binned into four temperature classes. Temperature classes were determined by binning leaf temperature data while excluding empty bins (e.g., there were no measurements between 12°C and 15°C), and separate $g_s$ versus $\ln D$ relationships were fit for each leaf temperature class (Figures 1a and 1b). In each temperature class, $g_s$ at a $D$ of 1.6 kPa (the $D$ used to model seasonal carbon gain; see below) was estimated using the modeled $g_{sref}$ and slopes (Figures 1c and 1d). The final Jarvis-Oren model described the change in $g_s$ at a $D$ of 1.6 kPa with a change in leaf temperature for both treatments, and was in the form:

$$g_s = m_3 (1/T_{leaf}) + b_3$$

where $m_3$ is a treatment specific parameter, $b_3$ is a minimum $g_s$, and $T_{leaf}$ is leaf temperature in °C (Table 2 and

| Table 2. Comparison of Parameters and Parameter Values Used for Each $g_s$ Model$^*$ |
|---------------------------------|-----|-----|
| Parameter (units) | AT | HT |
| $g_s$ (mol m$^{-2}$ s$^{-1}$) | 0.14 | 0.050 |
| $C_i/C_a$ | 0.70 | 0.70 |
| **Ball-Berry** | | |
| $m_1$ (mol m$^{-2}$ s$^{-1}$) | 2.851 | 8.099 |
| $b_1$ (mol m$^{-2}$ s$^{-1}$) | 0.0974 | 0.0336 |
| **Leuning** | | |
| $m_2$ | 5.218 | 8.223 |
| $b_2$ (mol m$^{-2}$ s$^{-1}$) | 0.0565 | 0.0106 |
| **Jarvis-Oren** | | |
| $m_3$ | -1.4541 | -3.0566 |
| $b_3$ (mol m$^{-2}$ s$^{-1}$) | 0.1532 | 0.2175 |
| **Linear Optimization** | | |
| $s$ | 0.061 | 0.038 |
| $\lambda$ (mol mol$^{-1}$) | 0.70 | 0.70 |

$^*$All data used to derive parameter values are from Way and Sage [2008a, 2008b], except for $\lambda$, which is derived from equation (9). AT, ambient temperature treatment; HT, high temperature treatment.

It should be noted that this model maintains a saturating increase of $A$ with $C_a$ and preserves the linear relationship between $g_s$ and $(C_a - \Gamma_s)$ in the models described next.
Figure 1. Response of stomatal conductance ($g_s$) to variation in vapor pressure deficit ($D$) measured at saturating light and different leaf temperature classes (symbols: 9°C–12°C, black; 15°C–22°C, dark gray; 25°C–32°C, light gray; >35°C, white) on black spruce grown at day/night temperatures of (a) 22/16°C (ambient temperature, AT) and (b) 30/24°C (high temperature, HT). Inset of Figure 1b compares the measured and modeled sensitivities ($m_1/b_1$) for each temperature class. Based on the relationships in Figures 1a and 1b, the effect of a change in leaf temperature on (c) $g_{sref}$ ($g_s$ at a reference $D$ of 1 kPa); (d) $\delta g_s/\delta \ln D$; and (e) $g_s$ at a $D$ of 1 kPa (for b inset: AT, upward triangles; HT, downward triangle. For Figures 1c–1e: AT, open symbols, dashed lines; HT, filled symbols, solid lines).

Equation (7) was used for the Jarvis-Oren model in the model comparison.

Since there are expected relationships between $\delta g_s/\delta \ln D$ and $g_{sref}$ in the Jarvis-Oren model [Oren et al., 1999; Kim et al., 2008], the expected sensitivity (defined as the ratio of $\delta g_s/\delta \ln D$ to $g_{sref}$) was modeled for the four temperature classes for each treatment as a test of the Jarvis-Oren model performance. Within each temperature class in each treatment, the relationship between $g_{sref}$ and $\ln D$ was determined using mean $g_s$ values from each treatment, a boundary layer conductance ($g_{bl}$) of 0.936 mol m$^{-2}$ s$^{-1}$ to account for differences between needle and shoot boundary layers in black spruce [Rayment et al., 2000], and all measurements where $D$ was within two standard deviations of the mean $D$ to exclude outliers from the $D$ range data.

The sixth model was a Linear Optimization model, based on the theory that $g_s$ autonomously maximizes leaf carbon gain for a given water loss [Givnish and Vermeij, 1976; Cowan, 1978; Cowan and Farquhar, 1977; Hari et al., 1986]. In linearizing the photosynthetic CO$_2$ curve of equation (1),
is a conversion factor needed to convert \( \text{dt} \) molecules to biomass carbon, Mortazavi et al., Manzoni et al., Ron the functions relating \[1999\] used in the Jarvis-Oren model \[f; \] scales linearly with reduction in the Leuning \( D \) is autotrophic respiration (Table 2), flux rates were modeled for a \( D \) increases linearly with atmospheric \( C \) is the total seedling \( R \) are the beginning and end times of the \( t = 0.6 \) (short dashed line), \( t = 1 \) kPa. The gray box indicates a \( D \) of 1.6 kPa, the constant \( D \) used in the analysis.

Katul et al. [2009, 2010] derived analytical expressions for \( g_s, C_i/C_a \) and \( A \) that are here modified to account for \( \Gamma_* \) (see Appendix A):

\[
g_s = \frac{a_1}{a_2 + \alpha C_a} \left[ -1 + \frac{(C_a - \Gamma_*)}{a\lambda D} \right] + g_0 \tag{8}
\]

\[
\frac{C_i}{C_a} = 1 - \sqrt{\frac{a\lambda D (C_a - \Gamma_*)}{C_a}} \tag{9}
\]

\[
A = \frac{a_1 (C_a - \Gamma_*)}{a_2 + \alpha C_a} \left[ 1 - \frac{a\lambda D}{(C_a - \Gamma_*)} \right] \tag{10}
\]

where \( s \) is, as before, the long-term mean \( C_i/C_a \), \( a \) is the relative diffusivity of water compared to \( \text{CO}_2 (=1.6) \), \( g_s \) is nighttime stomatal conductance, and \( \lambda \) is a species-specific cost parameter for water loss in units of carbon (also known as the marginal water use efficiency, where \( \lambda = \delta A/\delta E \)). We set \( s \) to 0.7 based on the measured mean daytime \( C_i/C_a \) for both AT and HT seedlings measured at daytime growth conditions (see Constant \( C_i/C_a \) model) and used the mean \( g_s \) measured in the dark at ambient \( \text{CO}_2 \) concentrations, and at temperatures within \( \pm 6^\circ \text{C} \) of nighttime growth temperatures for each treatment to estimate \( g_{s0} \). Our value for \( \lambda \) was derived by rearranging equation (9) and assuming that \( C_a - \Gamma_* \approx 1 \), such that \( \lambda \) scales linearly with \( C_a \) (Table 2), consistent with the studies discussed by Katul et al. [2010] and Manzoni et al. [2011].

The models, while different, have a number of underlying similarities. Operationally, the Linear Optimization model approximately retains the \( D \) response from Oren et al. [1999] used in the Jarvis-Oren model [Katul et al., 2009]. The Linear Optimization model also retains the quasi-linear correlation between \( g_s \) and \( A/(C_a - \Gamma_*) \) noted in the numerous gas exchange data sets used to derive the Ball-Berry and Leuning models, such that this relationship can be seen as an ‘emergent’ property of the Linear Optimization model, provided \( \lambda \) increases linearly with atmospheric \( \text{CO}_2 \) (as shown by Katul et al. [2009, 2010]). As well, the Leuning and Linear Optimization models only differ in their nonlinear functional dependence on \( D \), which is most amplified for \( D < D_m \), where the \( D \) reduction in the Leuning model is quasi-linear, but that of the Linear Optimization model exhibits significant nonlinearity (Figure 2; see Appendix A). Equation (9) from the Linear Optimization model is also consistent with studies showing that \( C_i/C_a \) decreases nonlinearly with increasing \( D \) [Wong and Dunin, 1987; Mortazavi et al., 2005], which differs from the linear decline in \( C_i/C_a \) predicted by the Leuning model [Katul et al., 2000].

2.5. Testing the Stomatal Conductance Models With the Carbon Budget Closure

\[
\frac{dB}{dt} = CF[LA(t) \times A(t)] - (R_E + R_C); \tag{11}
\]

which can be re-arranged to yield:

\[
B(t_f) - B(t_i) = \int_{t_i}^{t_f} \{CF[LA(t) \times A(t)] - (R_E + R_C)}dt; \tag{12}
\]

where \( t \) is time, \( t_i, t_f \) are the beginning and end times of the growing season, \( CF \) is a conversion factor needed to convert \( \text{CO}_2 \) molecules to biomass carbon, \( LA \) is the total seedling leaf area (which evolves over time), \( R_E \) is autotrophic respiration, and \( R_C \) is the construction cost.

With regards to modeling \( A(t) \), diurnal leaf responses of \( g_s, C_i/C_a \) and net \( \text{CO}_2 \) flux rates were modeled for a

![Figure 3](image-url)
24 h period with temperature and light conditions representative of the growth conditions in the work by Way and Sage [2008b]; mean leaf temperatures based on 24 h thermocouple readings, 14/10 day/night photoperiods, 800 μmol photons m⁻² s⁻¹ PPFD, 400 μmol mol⁻¹ CO₂, and a constant D of 1.6 kPa (Figure 3). For the Constant gᵣ model, gᵣ was set and equations (1) and (2) were solved for A and Cᵣ. For the Constant Cᵣ/Cₑ model, we set Cᵣ/Cₑ to its measured mean value and used equations (1) and (2) to predict A. For the Ball-Berry, Jarvis-Oren, Leuning, and Linear Optimization models, gᵣ, Cᵣ, and A were solved based on equations (1) and (2) and the gᵣ equation for each of the four models. Net CO₂ assimilation rates (Aₜₜₑₓ) per unit leaf area were calculated for 10 min blocks using temperature-corrected Vₑₘₐₓ and Γ* (Table 1). These rates were then converted to net carbon exchange (g C m⁻²) for each 10 min interval of the light period and summed for a 24 h period for each gᵣ model.

[20] With regard to estimating Bᵣ(tᵣ), Bᵣ(tᵣ), and L(Aᵣ), data on seedling growth were taken from AT and HT black spruce [Way and Sage, 2008a, 2008b]. P. mariana seeds are small (thousand-seed mass = 1.12 g [Wang and Berjak, 2000]), so Bᵣ(tᵣ) was ignored. Measured changes in leaf, stem, and root mass for each treatment were used to fit exponential growth trajectories for each pool over a 205 day timeframe (the duration of the experiment in the work by Way and Sage [2008b]), allowing us to estimate seasonal growth in each carbon pool, as well as total biomass (B). Using this 205 day growth season, the time evolution of LA in equation (12) was computed by converting calculated daily leaf mass for each treatment to daily leaf area, using measured specific leaf areas for each treatment. As in equation (12), the computed daily leaf area was multiplied by the modeled net carbon exchange per unit leaf area (in g C m⁻²) for each gᵣ model to calculate daily carbon gain (in g C). Daily carbon gain values over the 205 days were then summed to obtain total leaf carbon gain over the growth season for each gᵣ model in each temperature treatment.

[21] The Bₑ was estimated from calculated daily leaf, stem, and root masses as above, as well as daily stem height and diameters calculated from exponential fits to measured changes in height and diameter for each treatment. Root respiration was calculated as daily root mass multiplied by root respiration rates from black spruce seedlings grown at either 24/18°C or 30/24°C day/night temperatures [Tjoelker et al., 1999] over a 24 h period. Shoot dark respiration was calculated based on measured leaf and stem mass regressions, and shoot dark respiration rates for black spruce seedlings grown at either 24/18°C or 30/24°C day/night temperatures [Tjoelker et al., 1999] over the night period, scaled to measured leaf temperatures with Q₁₀ values from the same study. Published daytime stem respiration rates were on a surface area basis [Acosta et al., 2008]. Hence, daily stem surface area was estimated using the regressions for shoot height and stem diameter and multiplying height by 2/3 diameter to account for stem taper. Because gas exchange was measured on branches and not just leaves, branch and leaf day respiration are already accounted for in values of Aᵣₑₓ.

[22] To estimate carbon in live biomass at tᵣ (Bₑᵣ), leaf, stem, and root mass values for day 205 were multiplied by measured %C values for leaves (47%) or values from the literature for stems (48%) [Iivonen et al., 2006; Kaakinen et al., 2009; Kostiainen et al., 2009] and fine roots (45% [Iivonen et al., 2006; Jackson et al., 2009]). Construction costs of biomass (Rₑ) were estimated as 1.5 g glucose g⁻¹ dry mass [Niinemets, 1997].

[23] Total seasonal carbon gain from each gᵣ model was compared to the summed carbon costs of biomass, construction costs, and respiration as (Cₑᵢₑₓ – Cₑₑᵢₑₓ)/Cₑₑᵢₑₓ to estimate closure of the seedlings’ carbon budgets for each growth temperature, where:

\[
Cₑₑᵢₑₓ = Bₑᵣ + \int_0^{tᵣ} \{Rₑ(\tau)\} d\tau;
\]

\[
Cₑᵢₑₓ = B(0) + \int_0^{tᵣ} \{CF[LA(\tau) \times A(\tau)]\} d\tau.
\]

While the daily leaf area used to drive seasonal carbon gain was derived from regressions of leaf mass measurements (and seasonal specific leaf area), carbon costs relied on end of season total biomass, with leaf mass only contributing to carbon costs through the daily shoot mass regressions used to calculate shoot dark respiration.

[24] Because seasonal carbon costs were our benchmark for model performance, we estimated potential error in these costs by determining the minimum and maximum seasonal carbon costs from our measured biomass and measured and literature-based respiration rates. Although the modeled growing season was 205 days long, seedling biomass was measured for three independent replicates of the temperature acclimation experiment 197, 205 and 210 days after germination [Way and Sage 2008a, 2008b, unpublished data, 2007]. For each growth temperature, the smallest, youngest seedlings (197 days old) were used to estimate the lower value of mean seasonal biomass; the biggest, oldest seedlings (210 days old) were used to generate an upper value of seasonal biomass. Construction costs for these lower and upper bounds of seedling mass were determined as above. Since construction costs were based on biomass, biomass values accounted for 61%-79% of the minimum and maximum seasonal carbon cost estimates. Potential errors in respiratory costs were accounted for by using minimum and maximum estimates from the literature for stem day respiration [Acosta et al., 2008] and the lowest and highest shoot dark respiration rates measured; root respiration was not varied, but accounted for <10% of total seasonal carbon costs. These lowest respiration costs were summed with the smallest seasonal biomass and construction costs to provide a minimum seasonal carbon cost estimate; the highest respiration rates were combined with the largest biomass and construction costs to generate an upper estimate of seasonal carbon costs. However, the estimates will inherently overestimate error in our modeled seasonal carbon costs. While all of the biomass measurements represent “end of season” biomass, plant growth is exponential; seedlings harvested 197 days after germination will be smaller than if they had grown for 205 days (the growth season modeled here), while the mass of seedlings grown for 210 days will be much greater than for those same seedlings 205 days after germination. Because these error estimates will be high, we also used a second, more conservative indicator of model success, by
testing which $g_s$ models generated seasonal carbon gain values within 10% of the modeled seasonal carbon losses.

2.6. Statistics

[25] Regressions and ANOVAs were performed in JMP 8.0.2 (SAS, Cary, North Carolina, USA).

3. Results

[26] The rate of seedling growth over time was consistent between experiments [Way and Sage, 2008a, 2008b, unpublished data, 2007], despite differences in maximum light levels and humidity conditions during growth, caused by greenhouse versus growth chamber growth conditions (Figure 4). Because seedling growth is exponential and the height and biomass data were best described, and described well, by exponential functions, we used exponential regressions to estimate seedling growth and mass. The most extensively measured parameter was shoot height, which was well-described with a single exponential growth function for each temperature treatment, with coefficients of determination ($r^2$) of 0.85 to 0.90 for AT and HT seedlings, respectively ($p < 0.0001$ for both; Figure 4a). Total biomass, leaf mass, stem mass, and root mass in each treatment also developed along similar exponential growth trajectories over time between experiments, with $r^2$ values ranging from 0.91 to 0.99 ($p < 0.022$ for all, Figures 4b–4d).

[27] The Constant $C_i/C_a$, Ball-Berry, and Leuning models and the Linear Optimization model’s $D$ reduction function all produced highly significant fits between the functions used to model $g_s$ and the measured $g_s$ values, and all provided a better fit to the HT data set (Figure 5). Coefficients of determination ($r^2$) for the AT and HT data, respectively, across the models were: Constant $C_i/C_a$, 0.04 ($p = 0.0013$) and 0.25 ($p < 0.0001$; Figure 5a); Ball-Berry, 0.04 ($p = 0.0019$) and 0.30 ($p < 0.0001$; Figure 5b); Leuning, 0.28 and 0.48 ($p < 0.0001$ for both; Figure 5c); and Linear Optimization $D$ function, 0.24 and 0.49 ($p < 0.0001$ for both; Figure 5d). Growth at elevated temperatures increased $m_1$ and $m_2$ in the Ball-Berry and Leuning models, respectively (Table 2).

[28] In building the final Jarvis-Oren model equation, the relationship between $g_s$ and ln$D$ was related to leaf temperature in both treatments ($r^2$ s of 0.22–0.73, $p < 0.0001$ for all; Figures 1a and 1b). While higher measurement temperatures often corresponded to higher $D$, within each temperature class, $g_s$ was measured over a range of $D$ and there was no pattern between temperature and $D$ (data not shown). The measured sensitivity (i.e., the ratio of $\delta g_s/\delta \ln D$ to $g_{ref}$) of the AT data in each temperature class was consistent with the sensitivity modeled using the approach of Oren et al. [1999] (Figure 1b inset). However, the measured HT sensitivity deviated from modeled expectations at low leaf temperatures; there was good agreement between measured and modeled values at 30°C and 40°C, but measured sensitivity was slightly higher than expected at 20°C and much higher than predicted at 10°C leaf temperatures (Figure 1b inset). Values for $g_{ref}$ increased with rising leaf temperatures in the HT treatment ($r^2 = 0.99, p < 0.005$), and peaked at moderate leaf temperatures (∼20°C) for AT seedlings (Figure 1c); because the response of AT $g_{ref}$ to temperature could not be significantly described for the four data points ($p > 0.05$), we selected the curve fit to both maximize the coefficients of determination and minimize the number of parameters ($r^2 = 0.74$, parameter n = 3). The slope of the relationship between $g_s$ and ln$D$ increased with leaf temperature for both treatments, such that $g_s$ was more sensitive to variation in $D$ at low leaf temperatures than at warm temperatures ($r^2 = 0.88–0.94$, $p = 0.03–0.06$; Figure 1d). Values of $g_s$ at a $D$ of 1.6 kPa showed a similar response to rising leaf temperatures in both treatments ($r^2 = 0.97–0.99, p < 0.016$ for both; Figure 1e).
Daily courses of modeled $g_s$, $C_i/C_a$, $A_{net}$ and carbon gain for each treatment varied considerably between models (Figure 6). For AT seedlings, the Constant $C_i/C_a$ and Leuning models predicted the highest daytime $g_s$, while the Ball-Berry, Linear Optimization, and Jarvis-Oren models had similar, lower $g_s$ values (Figure 6a). While the Leuning model also predicted relatively high $g_s$ for the HT seedlings, the Jarvis-Oren model generated the highest $g_s$ values, and although the Constant $C_i/C_a$ model predicted higher $g_s$ in the AT treatment, it generated the lowest $g_s$ values in the HT treatment (Figure 6b). In both the AT and HT scenarios, the Linear Optimization model consistently predicted the highest rates of net photosynthesis and daily carbon gain (Figures 6c–6e). Although there were not many gas exchange measurements at the modeled daytime conditions, model outputs can be compared to a subset of the data (n = 45 for AT and 15 for HT leaves) collected at saturating light, ambient CO$_2$ and leaf temperatures near growth conditions (25.2 ± 0.2°C for AT leaves and 35.3 ± 0.1°C for HT leaves, means ± SE). In this data set, $A_{net}$ was $7.5 ± 0.3$ for AT and $4.8 ± 0.4$ μmol m$^{-2}$ s$^{-1}$ for HT seedlings, while $g_s$ was $0.11 ± 0.01$ and $0.09 ± 0.01$ μmol m$^{-2}$ s$^{-1}$ for AT and HT leaves, respectively (means ± SE). These measured $g_s$ values were similar to model outputs for both treatments, with models predicting both higher and lower $g_s$, while the measured $A_{net}$ values were slightly lower than modeled $A_{net}$ for AT leaves and similar to the lower values modeled in the HT leaves by the Ball-Berry model (Figure 6). The mean daily WUE ($A/E$) predicted by each model was also compared between models, to determine whether models were generating realistic combinations of $g_s$ and $A_{net}$. Values for modeled WUE at a $D$ of 1.6 kPa ranged from 3.8–6.4 μmol CO$_2$/mmol H$_2$O in AT leaves and 2.8–10.7 μmol CO$_2$/mmol H$_2$O in HT leaves (Table 3); mean values for the subset of measured data near growth conditions and $D$ near 1.6 kPa were 5.4 and 3.3 μmol CO$_2$/mmol H$_2$O for AT and HT spruce, with values ranging from 2.1–10.4 μmol CO$_2$/mmol H$_2$O. 

Modeled seasonal carbon costs for a 205 day growing season (based on exponential biomass growth, respiration, and construction costs as described above) were in the middle of the range of seasonal carbon costs estimated from total biomass harvested 197 and 210 days after germination, along with minimum and maximum respiration estimates. Estimates of carbon costs based on 197 day old seedlings were 35% lower than modeled 205 day carbon costs, while maximum seasonal carbon cost estimates from 210 day old seedlings ranged from 28%–35% greater than modeled carbon costs (Figure 7a). Since exponential growth produces rapid changes in total mass (Figure 7a inset), the actual seasonal carbon costs for seedlings that are 205 days old (as modeled here) will be larger than those estimated from seedlings that were harvested 197 days after germination and will be smaller than those derived from seedlings that are five days older (210 days old). Because our estimates of minimum and maximum seasonal carbon gain from measured seedlings were expected to be overestimates, we also used our modeled carbon costs with a 10% error as a more stringent estimate of the ability of the $g_s$ models to describe the carbon gain of both treatments (Figure 7b).
Models varied considerably in their ability to estimate the carbon gain necessary to match the seasonal carbon costs and in the consistency of their performance between the growth temperatures (Figures 7a and 7b). All of the models were able to predict a sufficiently high carbon gain to match the minimum seasonal carbon costs estimated from each treatment (i.e., that of 197 day old seedlings; Figure 7a). However, only the Leuning and Linear Optimization models successfully described the carbon budget of both treatments within 10% of the best estimate of total carbon costs for 205 day old seedlings (Figure 7b). The Constant \( g_s \) and Jarvis-Oren models could predict the carbon gain of either

**Figure 6.** Diurnal time course of (a, b) stomatal conductance (\( g_s \)), (c, d) 1 minus the ratio of intercellular to ambient CO\(_2\) (\( C_i/C_a \)), (e, f) net CO\(_2\) assimilation rates (\( A_{net} \)), and (g, h) carbon gain (or loss, for dark respiration) for various \( g_s \) models for black spruce grown at 22/16°C (ambient temperature; AT) or 30/24°C day/night temperatures (high temperature; HT, black circles). Models: Constant \( g_s \) (magenta); Constant \( C_i/C_a \) (cyan); Ball-Berry (yellow); Leuning (black); Jarvis-Oren (green); Linear Optimization (red); dark respiration (\( R_{dark} \), blue). Note that there are no \( g_s \) estimates for the Constant \( C_i/C_a \) model in Figures 6a and 6b.

**Table 3. Comparison of Mean Daily Instantaneous Water Use Efficiencies for Each \( g_s \) Model**

<table>
<thead>
<tr>
<th>Model</th>
<th>AT (±SD)</th>
<th>HT (±SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ( g_s )</td>
<td>4.3 ± 1.7</td>
<td>7.4 ± 0.2</td>
</tr>
<tr>
<td>Ball-Berry</td>
<td>5.5 ± 2.0</td>
<td>10.7 ± 0.3</td>
</tr>
<tr>
<td>Jarvis-Oren</td>
<td>6.3 ± 0.4</td>
<td>2.8 ± 0.3</td>
</tr>
<tr>
<td>Leuning</td>
<td>3.8 ± 1.0</td>
<td>3.8 ± 0.1</td>
</tr>
<tr>
<td>Linear Optimization</td>
<td>6.4 ± 1.1</td>
<td>7.2 ± 0.1</td>
</tr>
</tbody>
</table>

\( ^{a} \)WUE measured in \( \mu \)mol CO\(_2\)/mmol H\(_2\)O. AT, ambient temperature treatment; HT, high temperature treatment.
the AT or HT seedlings, respectively, for this more stringent criterion, but could not equally estimate seedling carbon gain for the second treatment (Figure 7b). In contrast, the Constant \( \frac{C_i}{C_a} \) and Ball-Berry models were unsuccessful in capturing the carbon fluxes for either group of trees within 10% of modeled carbon costs.

4. Discussion

Seasonal carbon costs were used as the benchmark for \( g_s \) model performance, and since carbon costs were mainly determined by biomass (directly and also indirectly through construction costs), we assessed potential error in our biomass values by using measured seedling mass at the end of the growing season in three different replicate experiments. While this produced a twofold range of values that encompassed our modeled carbon cost values (Figure 7a), this is certainly an overestimate of the uncertainty of these measurements. The minimum carbon cost was estimated from plants harvested eight days earlier than our modeled growing season length of 205 days, and would be higher with an extra week of exponential growth. Similarly, the maximum estimated carbon costs were derived from seedlings harvested five days later than our 205 day modeled growing season.
length and thus overestimate seasonal growth and carbon costs.

[33] Since it is implausible to collect measurements of \( g_s \) responses to environmental variation for all species of interest, models of \( g_s \) are needed. One way to judge model performance is to compare measured versus modeled \( g_s \), as shown in Figure 5. However, our interest was in whether \( g_s \) models could be tested on a longer time scale. Of the six \( g_s \) models tested here, only the Leuning and Linear Optimization models captured the carbon gain for both treatments within 10% of our carbon loss values. The Constant \( g_s \) model met the 10% threshold criteria for AT seedlings and the Jarvis-Oren model for HT seedlings, while the Constant \( C_i/C_a \) and Ball-Berry models underestimated carbon gain in both treatments by more than 10%, with the Ball-Berry model underestimating carbon gain by more than 20% for the HT data. Contrasting the results between the \( g_s \) models and treatments allows us to narrow down the reasons for the variation in long-term model performance.

[34] Neither the Constant \( g_s \) model nor the Constant \( C_i/C_a \) model performed very well in closing the carbon budgets, although the Constant \( g_s \) predicted carbon gain just within 10% of estimated carbon costs for AT seedlings. While conifer stomata tend to be slow to respond to changes in their environment, thus dampening the extent of their response to stimuli [Watts and Neilson, 1978; Ng and Jarvis, 1980], these responses must still be included in models to fully capture the plant’s carbon dynamics. As well, spruce from both the AT and HT treatments had a similar mean \( C_i/C_a \) of 0.7, implying that this might represent an optimal balance between \( g_s \) and \( A_{net} \) across growth temperatures [Wong et al., 1979]. However, the Constant \( C_i/C_a \) model could not satisfactorily predict carbon gain in either treatment, thereby disputing the idea of an optimal \( C_i/C_a \).

[35] The Ball-Berry and Leuning models have similar forms, but the Ball-Berry model predicts \( g_s \) based on RH responses and the Leuning model from responses to \( D \) (compare equations (4) and (5)). While the Leuning model described both AT and HT carbon gain well, the Ball-Berry model performed poorly with both sets of data, due to its predictions of very low \( g_s \). Given the low explanatory power of the Ball-Berry model on the \( g_s \) data (Figure 5b), its inability to predict \( g_s \) and thus carbon gain, is not surprising. However, the differences in the carbon balance predictions between the Ball-Berry and Leuning models also imply that \( D \) is a better predictor of \( g_s \) than relative humidity, a result with empirical support in the physiological literature [Apfelho and Jarvis, 1991] and in a previous comparison of the Ball-Berry and Leuning models on plants grown at elevated temperatures [Nijs et al., 1997]. Indeed, the models that performed best overall in our analysis (Leuning, Linear Optimization as well as the Jarvis-Oren model for HT seedlings), all related \( g_s \) to \( D \). While the conceptual linkage of \( g_s \) to \( D \) is similar in the three models, the forms of the relationship are different, with important implications for predicting \( g_s \) as \( D \) rises (Figure 2). At low \( D (<1 \text{ kPa}) \), the Linear Optimization model and the function used in the Jarvis-Oren model [from Oren et al., 1999] are similar, but contrast with the Leuning model. When \( D \) is high (>3 kPa), the Leuning and Linear Optimization formulations become quasi-linear, while the Oren et al. [1999] function continues to decline. The distinctions between these models, especially at high \( D \), will be important in a warming world, since climate warming is not expected to significantly alter air relative humidity, but should increase \( D \) because of increases in saturation vapor pressure [Kumagai et al., 2004].

[36] There was good agreement between the measured and expected sensitivities (the ratio of \( \delta g_s/\delta \ln D \) to \( g_{ref} \)) of the Jarvis-Oren model in AT seedlings in all temperature classes and for HT seedlings measured at moderate to high leaf temperatures, demonstrating that this data was well-described by the relationships derived by Oren et al. [1999]. However, HT sensitivity was much higher than expected at leaf temperatures near 10°C. While the AT data was captured within each temperature class by the Jarvis-Oren model, the pattern of responses of \( g_{ref} \) and \( \delta g_s/\delta \ln D \) to leaf temperature in the AT treatment were less reasonable. The estimation of AT daily carbon gain operated in the range of leaf temperatures (20°C–25°C) where, based on modeled responses to leaf temperature, \( g_{ref} \) would be underestimated and \( \delta g_s/\delta \ln D \) would be overestimated. The net result of these two biases was that \( g_s \) predictions at 1.6 kPa in this temperature range were too low, reducing predicted \( C_i \) and \( A_{net} \), and generating the Jarvis-Oren model’s underestimation of seasonal carbon gain in AT trees. In contrast, the response of \( g_s \) to \( \ln D \) was not well-captured by the Jarvis-Oren model for HT spruce at low leaf temperatures, as seen by the difference between measured and expected sensitivity at 10°C, leading to predictions of negative \( g_s \) at 10°C and 1.6 kPa (Figure 1e). However, the Jarvis-Oren model performed well at the warmer temperatures where HT leaves were operating (30°C–40°C). Because the Jarvis-Oren model predicted both \( g_{ref} \) and \( \delta g_s/\delta \ln D \) well at leaf temperatures of 30°C–35°C, the model produced good closure of the carbon budget in this treatment.

[37] The Linear Optimization model predicted both AT and HT carbon gain well, consistent with the theory that stomata regulate \( g_s \) to maximize photosynthetic carbon gain while minimizing water loss [Givnish and Vermeij, 1976; Cowan, 1978; Cowan and Farquhar, 1977; Hari et al., 1986; Arneth et al., 2002; Konrad et al., 2008; Katul et al., 2009, 2010]. This and the Leuning model were the only models to accurately capture seasonal carbon gain within 10% of modeled costs in both treatments. The similarity in their success is not surprising, as the Linear Optimization model resembles the Leuning model except that their \( D \) reduction functions are not identical (see Appendix). And while the Leuning model generated instantaneous WUE values that most closely matched our measured values, instantaneous WUE in woody \( C_3 \) species varies from 1.0–7.8 μmol mol⁻¹ (with the highest value being for Picea glauca) [Yoo et al., 2009] and from 2–10 μmol mol⁻¹ in our data, so only the Ball-Berry model produced WUE values outside our measured range. Our finding that the Leuning and Linear Optimization models both perform well contrasts with Nijs et al. [1997], who found that an approach based on a form of optimization performed more poorly than either the Ball-Berry or Leuning models in plants grown under both ambient and future \( \text{CO}_2 \) and temperature conditions. It should be emphasized that Nijs et al. [1997] evaluated a form of optimization theory based on maximizing instantaneous WUE, which is not comparable with the constant marginal water use efficiency used in the Linear Optimization model, although the two water use efficiencies
can be theoretically linked. As shown in the Appendix, the flux-based instantaneous WUE is not an intrinsic plant property, but varies with external environmental conditions. In the context of the Linear Optimization model, WUE increases linearly with increasing \( C_i \) and, perhaps more pertinent here, declines nonlinearly (as \( D^{-1/2} \)) with increasing \( D \).

[38] The Linear Optimization model can explain \( g_s \) patterns in plants grown at different CO\(_2\) concentrations and exposed to various water stress levels [Kutul et al., 2009, 2010; Manzioni et al., 2011]. Our results add to the conclusion that this approach is useful for dealing with not only current vegetation, but also plants under future climate change scenarios. The Linear Optimization model uses a species-specific \( \lambda \), which was held constant in both growth temperatures; the ability of this same \( \lambda \) to close the carbon budget for both treatments suggests that \( \lambda \) does not vary appreciably with warming. Other environmental conditions can alter \( \lambda \); elevated CO\(_2\) increases \( \lambda \) in a predictable way [Kutul et al., 2010; Manzioni et al., 2011], but variations in \( \lambda \) with water availability are more complex. Soil volumetric water content between 15% and 30% had almost no effect on \( \lambda \) in Scots pine (Pinus sylvestris), and while decreasing soil water content can increase \( \lambda \) sevenfold, this only occurred at extremely stressful conditions [Kolari et al., 2009]. Recent work has shown that \( \lambda \) varies with soil water availability, and that the shape of this response curve differs between plant functional types [Manzioni et al., 2011]. While we found no need to vary \( \lambda \) between treatments, more research on the effect of growth temperature on \( \lambda \) is needed to make a definitive statement on whether \( \lambda \) will change with rising air temperatures.

5. Conclusions

[39] Large-scale modeling efforts, such as coupled vegetation-climate, hydrologic and ecological models, currently rely on semi-empirical \( g_s \) models. In fact, the Ball-Berry model was used in global climate models as early as 1995 [Sellers et al., 1995], and more detailed biosphere-atmosphere models primarily employ the Ball-Berry formulation [Baldocchi, 1997; Anderson et al., 2000; Luo et al., 2001; Reichstein et al., 2003; Blanken and Black, 2004] and Leuning models tested here [Whitehead et al., 2001; Keenan et al., 2010]. We show that the semi-empirical Leuning and the Linear Optimization-based models performed best for spruce grown at ambient and elevated temperatures, both in capturing measured \( g_s \) on a short timescale and carbon gain on a longer, seasonal timescale. Since optimization theory does not use a priori relationships between \( g_s \) and environmental conditions, but focuses on ecological theories to predict them, these models are likely to hold true across future conditions where empirical data is scarce. If a semi-empirical model is to be used in large-scale modeling, our results support the use of the Leuning model over the Ball-Berry model, particularly in vegetation modeled under future climate scenarios [see also Nijs et al., 1997]. However, changing the Leuning model \( D \) reduction function from \( 1/(1 + D/D_m) \) to \( D^{-1/2} \) is preferable, for its consistency with the Oren et al. [1999] function (tested across many scales and species) and the advantage of reducing the number of empirical parameters needed to model \( g_s \).

Evaluating the impact of this change on climate model outputs, particularly under future climates, would be a first step toward testing the robustness of current predictions of vegetation-climate feedbacks.

Appendix A: A Linearized Optimality Approach and Its Connection to the Leuning and Ball-Berry Models

[40] Linearizing the biochemical demand function in equation (1) results in a much simpler (and insightful) model for optimal \( g_s \). The linearization requires the assumption that the variability of \( C_i \) only marginally affects the denominator of equation (1), leading to an approximation of \( a_2 + C_i = a_2 + (C_i/C_a)C_a \approx a_2 + sC_a \). As a result,

\[
A = \frac{a_1(C_i - \Gamma_*)}{a_2 + sC_a}. \tag{A1}
\]

It must be stressed here that only in the denominator of equation (A1), \( s \) is treated as a model constant. Combining this linearized photosynthesis model with equation (2) results in an expression for \( C_i \) and \( A \) given by:

\[
C_i(g_s) = \frac{a_1\Gamma_* + a_2C_ag_s + C_s^2g_s}{a_1 + a_2g_s + g_sC_a}, \quad A(g_s) = \frac{a_1(C_s - \Gamma_*)g_s}{a_1 + g_s(a_2 + sC_a)}. \tag{A2}
\]

The objective function to be maximized by an autonomous leaf is to maximize photosynthesis for a given transpiration rate (\( E \)) resulting in:

\[
f(g_s) = A - \lambda \frac{g_s}{a_1 + g_s(a_2 + sC_a)} - \lambda a_2g_sD, \tag{A3}
\]

and upon differentiating this objective function with respect to \( g_s \), this yields:

\[
\frac{\partial f(g_s)}{\partial g_s} = -a_2\lambda D + \frac{a_1^2(C_s - \Gamma_*)}{[a_1 + g_s(a_2 + sC_a)]^2}. \tag{A4}
\]

Note that the convexity of \( f(g_s) \) versus \( g_s \) ensures that a maximum exist that can be determined by setting \( \partial f(g_s)/\partial g_s = 0 \) (i.e., maximum carbon gain for a given water loss). Solving for \( g_s \) results in:

\[
g_s = \frac{a_1}{a_2 + sC_a} \left[ -1 + \sqrt{\frac{C_s - \Gamma_*}{a_2\lambda D}} \right]. \tag{A5}
\]

Apart from the compensation point, this expression is identical to the one derived by Hari et al. [1986]. With this optimal conductance, the photosynthesis is given as:

\[
A = \frac{a_1(C_s - \Gamma_*)}{a_2 + sC_a} \left[ 1 - \sqrt{\frac{a_2\lambda D}{C_s - \Gamma_*}} \right]. \tag{A6}
\]

The above expression can be rearranged to yield:

\[
\frac{a_1}{a_2 + sC_a} = \frac{A}{(C_s - \Gamma_*)} \left[ 1 - \sqrt{\frac{a_2\lambda D}{(C_s - \Gamma_*)}} \right]. \tag{A7}
\]
so that

\[
 g_a = \frac{A}{(C_a - \Gamma*)} \left( 1 - \sqrt{\frac{C_a - \Gamma*}{a_1 \lambda D (C_a - \Gamma*)}} \right)
\]  

(A8)

If \( \lambda = \lambda_o C_a/\lambda_o \) where \( \lambda_o \) and \( \lambda_o \) are the intrinsic water use efficiency at the growth CO\(_2\) concentration and the growth CO\(_2\) concentration, respectively (such that the marginal water use efficiency increases linearly with increasing \( C_a \)), then

\[
 g_a = \frac{A}{(C_a - \Gamma*)} \left( 1 - \sqrt{\frac{C_a - \Gamma*}{a_1 \lambda D (C_a - \Gamma*)}} \right)
\]  

This functional form is identical to the Leuning model except that the vapor pressure deficit reduction function is \( D^{1/2} \) instead of \( \left( 1 + \frac{D}{D_o} \right)^{-1} \). Moreover, the sensitivity parameter of the Leuning model \( m_2 \) is given as

\[
 m_2 = \frac{a_1 \lambda_o \lambda_D}{a_1 \lambda_D} \left( \frac{C_o (1 - \Gamma*)/C_a}{a_1 \lambda_o} \right).
\]

Likewise, this Linear Optimization result is analogous to the Ball-Berry model if \( D^{1/2} \) is replaced by \( RH \).

[41] Based on the Linear Optimization results, the instantaneous water-use efficiency (WUE) can also be related to \( \lambda_o \), given as:

\[
 WUE = A \frac{a_1 \lambda_D}{C_a \sqrt{\lambda_o/C_a (a_1 \lambda_D/D^{1/2})}}.
\]  

(A10)

Note that if \( \lambda_o \) is constant, then WUE linearly increases with increasing \( C_a \) and nonlinearly decreases with increasing \( D \). Hence, unlike the marginal water use efficiency, the flux-based water use efficiency is not an ‘intrinsic’ plant property and it does vary with external environmental conditions.

[42] It is also instructive to compare the canonical form of the optimality solution in equation A5 with analytical solutions to the Ball-Berry or the Leuning models when intercepts \( b_1 \) and \( b_2 \) are small compared to \( g_a \). Upon combining equations (1) and (2) with equation (5), we obtain the following for the Ball-Berry model:

\[
 g_a \approx \frac{a_1 m_1 RH (1 + RH)}{(\Gamma* - c_o)} + \frac{m_1 RH (a_2 + C_a)}{(\Gamma* - c_o)}
\]  

(A11)

\[
 A \approx \frac{a_1 (1 + RH) (\Gamma* - c_o)}{a_2 + C_a}.
\]  

(A12)

Repeating the same analysis with equation (6) for the Leuning model, we obtain:

\[
 A \approx \frac{a_1 m_2 (1 + D/D_o)^{-1} (1 - (1 + D/D_o)^{-1})}{(\Gamma* - c_o)} + \frac{m_2 (1 + D/D_o)^{-1} (a_2 + C_a)}{(\Gamma* - c_o)}
\]  

(A13)

\[
 D \approx \frac{a_1 (1 + (1 + D/D_o)^{-1}) (\Gamma* - c_o)}{(\Gamma* - c_o) + m_2 (1 + D/D_o)^{-1} (a_2 + C_a)}.
\]  

(A14)

Naturally, the explicit dependence of \( g_a \) on the driving forces (RH and \( D \)), and thus temperature, differs across models. When comparing the models in Figures 5–7, the intercepts \( b_1 \) and \( b_2 \) were not ignored. While an analytical solution can be derived with intercepts \( b_1 \) and \( b_2 \) being finite, its mathematical form is too unwieldy for comparative purposes across models.

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\[
 L \approx \frac{a_1 (1 + (1 + D/D_o)^{-1}) (\Gamma* - c_o)}{(\Gamma* - c_o) + m_2 (1 + D/D_o)^{-1} (a_2 + C_a)}.
\]  

(A14)

References


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