A flow resistance model for assessing the impact of vegetation on flood routing mechanics

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Received 25 November 2010; revised 17 June 2011; accepted 28 June 2011; published 27 August 2011.

The specification of a flow resistance factor to account for vegetative effects in the Saint-Venant equation (SVE) remains uncertain and is a subject of active research in flood routing mechanics. Here, an analytical model for the flow resistance factor is proposed for submerged vegetation, where the water depth is commensurate with the canopy height and the roughness Reynolds number is sufficiently large so as to ignore viscous effects. The analytical model predicts that the resistance factor varies with three canonical length scales: the adjustment length scale that depends on the foliage drag and leaf area density, the canopy height, and the water level. These length scales can reasonably be inferred from a range of remote sensing products making the proposed flow resistance model eminently suitable for operational flood routing. Despite the numerous simplifications, agreement between measured and modeled resistance factors and bulk velocities is reasonable across a range of experimental and field studies. The proposed model asymptotically recovers the flow resistance formulation when the water depth greatly exceeds the canopy height. This analytical treatment provides a unifying framework that links the resistance factor to a number of concepts and length scales already in use to describe canopy turbulence. The implications of the coupling between the resistance factor and the water depth on solutions to the SVE are explored via a case study, which shows a reasonable match between empirical design standard and theoretical predictions.


1. Introduction

Equations describing the bulk flow velocity \( U_b \), such as Manning’s equation, are widely used in hydraulic engineering and surface hydrology, especially in the context of flood routing [Dooge, 1992; French, 1985; Hauser, 1996; Hornberger et al., 1998]. A range of ecological and environmental applications of such routing problems has renewed interest in the theoretical prediction of bulk flow velocity, especially in the field of ecohydrodynamics [Green, 2005]. These applications include urban wetland construction or restoration [Kadlec, 1994; Nepf, 1999], grass swale design to maximize pollution control in urban storm water runoff [Kirby et al., 2005], and linking tidal hydrodynamic forcing to flow and sediment transport over coastal wetlands [Christiansen et al., 2000; Koch and Gust, 1999; Leonard and Luther, 1995; Shi et al., 1995; Wang et al., 1993]. The behavior of extreme overbank flows, where floodplain vegetation may be inundated [Wilson and Hortritt, 2002], is also of interest due to its socio-economic implications and impact on large-scale biogeochemical cycling.

In flood routing, the flow mechanics are generally described by the Saint-Venant equations, which are mathematically “closed” by various approximations to the friction slope \( S_f \). In numerous applications, the \( S_f \) is inferred from Manning’s equation by assuming that the flow is locally steady and uniform [Ajayi et al., 2008; Moussa and Bocquillon, 2000, 2009; Parlange et al., 1981; Richardson and Julien, 1994; Singh and Woolhiser, 1976; Smith and Woolhiser, 1971; Tafur et al., 1993; Wang et al., 2002; Woolhiser and Liggett, 1967; Woolhiser, 1975; Yen, 2002]. The key parameter to be specified in this “closure” is the absolute surface roughness, which may be encoded in the momentum roughness height \( z_o \) or Manning’s roughness coefficient \( n \). These two roughness measures depend on the mean height of the roughness elements protruding into the flow \( D \) and their geometric arrangement. The convention is to assume that these two roughness measures are constant when the roughness Reynolds number \( z_o^* = u_o z_o / \nu \) is sufficiently large, where \( u_o \) is the friction velocity and \( \nu \) is the kinematic viscosity of water [Brutsaert, 1982; Gioia and Bombardelli, 2002; Huthoff et al., 2007]. For channels in which \( u_o \) can be determined from measured water depth \( H_o \) and bed slope \( S_b \), the \( z_o \) and \( n \) can be analytically linked to each other provided that \( H_o / D \) is large [Chen, 1991]. When \( H_o / D \) is of order unity, however, which is generally the case for flood routing over submerged vegetation, a priori specification of \( n \) pose unique conditions to the routing problem.
challenges. This approximates the situation for open channel flow over vegetated surfaces, where the canopy height \( H_c \) can be comparable to \( H_w \). For such cases, the flow resistance formulation is problematic, and to date, has resisted complete theoretical treatment thereby inviting the use of empirical formulations such as flow retardance curves [Kooven and Unny, 1969].

[4] For predicting the resistance parameters for flow through vegetation, an analysis of the mean momentum balance inside and just above dense and rigid canopies is proposed when \( H_w/H_c > 1 \). Because flood-routing over vegetated surfaces is often characterized by \( H_w/H_c > 1 \), the submerged vegetation case is considered here. The emergent vegetation case, while important in a number of scenarios, is beyond the scope. Next, the resulting mean velocity profile from the solution of the mean momentum balance is used to analytically describe \( n \) and the Darcy-Weisbach friction factor \( f \) as a function of three length scales. These length scales are \( H_w, H_c \) and \( L_c \) where \( L_c = (C_d \rho)^{-1} \) is the so-called adjustment length scale [Belcher et al., 2003], which is widely used in canopy turbulence to parameterize the loss of turbulent kinetic energy from advecting eddies due to their dissipation by drag elements [Belcher et al., 2003; Katul et al., 2004]. The \( L_c \) is dependent on a dimensionless drag coefficient \( C_d \), and the mean leaf area density \( a \approx \text{LAI}/H_c \), where LAI is the one-sided leaf area index.

[5] The predictions of the resistance factor from the proposed analytical model here are compared with a large number of flow experiments through vegetated canopies reported elsewhere [Poggi et al., 2009]. The model results show explicitly how \( n \) and \( f \) nonlinearly decrease with increasing \( H_w/H_c \) for a given \( L_c/H_c \), and with increasing \( L_c/H_c \) for a given \( H_w/H_c \). The implications of these nonlinear dependencies for flood routing dynamics are then explored. Using the one-dimensional Saint-Venant equation, the hydrographs and water depths along a stream covered uniformly with rigid vegetation with a constant \( L_c/H_c \) are computed using both a constant and a dynamically evolving \( n \) as a function of \( H_w/H_c \). The constant and dynamic \( n \) cases are contrasted in terms of the timing and modulation of the peak flow rate longitudinally.

2. Theory

[6] A brief review of the basic definitions and relationship between \( z_o \) and \( n \) (or \( f \)) for large \( H_w/H_c \) and for rigid vegetation is first discussed. An analysis of how this emerging picture is altered when \( H_w/H_c \) is of order unity is then presented. Even within this restrictive scope, \( n \) is shown to vary, at minimum, with \( H_w, L_c, \text{LAI}, \) and \( C_d \). The consequences of these alterations to \( n \) (or \( f \)) on the flood-routing mechanics are then analyzed via a case study using scaling arguments and numerical runs.

2.1. Basic Definitions and the Deep Layer Formulation

[7] The balance between frictional and gravitational forces in a wide rectangular channel whose control volume is shown in Figure 1a with \( H_w/H_c = 1 \) is discussed. In this analysis, it is assumed that (1) the bed slope angle \( \theta \) is small enough so that \( \sin(\theta) \approx \tan(\theta) = S_0 \), (2) the channel width is sufficiently large so that the hydraulic radius \( R_h \approx H_w \), (3) the flow is fully turbulent, statistically stationary and planar homogeneous, and (4) the surface can be treated as hydrodynamically rough \( (z_o^+ > 2) \). This last condition implies that the log-law with a constant \( z_o \) is a reasonable representation of the time-averaged velocity \( (U) \) profile over \( H_w \) as shown in Figure 1a. Under these conditions, the force balance between frictional forces at the canopy top and gravitational forces simplifies to (Figure 1a)

\[
\tau = \rho g (H_u - H_c) \sin(\theta) = \rho g (H_u - H_c) S_0,
\]

where \( \tau = \rho u^2 \) is the total stress approximated by the turbulent stress \( (\tau_{w}) \) shown in Figure 1a given that \( z_o^+ > 2 \). \( g \) is the gravitational acceleration, and \( \rho \) is the water density. With these definitions, equation (1) provides an estimate of \( u_e \) given as

\[
u_e = \sqrt{\frac{\tau}{\rho}} = \sqrt{g (H_u - H_c) S_0}.
\]

[8] The logarithmic mean velocity profile or variants on it for flexible canopies [Stephan and Gutknecht, 2002] is expressed as a function of \( z_o \) and \( u_e \) using

\[
\bar{U}(z) = \frac{u_e}{k} \ln \left( \frac{z - d}{z_o} \right),
\]

where overbar denotes time-averaging, \( z \) is height from the channel bottom, and \( k (= 0.4) \) is Von Karman’s constant [e.g., Izakson, 1937], and \( d \) is the zero-plane displacement (Figure 1a). The depth-averaged mean velocity or bulk velocity \( (U_b) \), is given as

\[
U_b = \frac{1}{H_w} \int_{z_o+d}^{H_w} \bar{U}(z) dz = \frac{u_e}{H_w} \int_{z_o+d}^{H_w} \ln \left( \frac{z - d}{z_o} \right) d(z - d)
\]

\[
= \frac{u_e}{H_w} \left[ (d - H_w + z_o) + (H_w - d) \ln \left( \frac{H_w - d}{z_o} \right) \right]
\]

\[
\approx \frac{u_e (H_w - d)}{H_w} \left[ -\ln(e) + \ln \left( \frac{H_w - d}{z_o} \right) \right] \approx \frac{u_e}{k} \ln \left( \frac{H_w}{e z_o} \right),
\]

where \( \ln(e) = 1 \) and for a deep-layer formulation, it was assumed \( H_w \gg z_o + d \). Rearranging equation (4) into a dimensionless form commonly used for flow resistance equations gives

\[
\frac{U_b}{u_e} = \frac{1}{k} \ln \left( \frac{H_w}{e z_o} \right).
\]

[9] Manning’s equation for a wide rectangular channel is

\[
U_b = \frac{1}{n} H_w^{2/3} S_0^{1/2}.
\]

When \( H_w \gg H_c \), and upon combining equations (2) and (6) gives

\[
\frac{U_b}{u_e} = \frac{H_w^{1/6}}{n^{1/3}}.
\]
Another flow resistance measure generally employed in flood routing is the Darcy-Weisbach friction factor \( f \), which can be related to \( n \) via

\[
\frac{U}{u_*} = \sqrt{\frac{8}{f} \frac{H_w^{1/6}}{n \sqrt{g}}}. \tag{8}
\]

To establish a simplified relationship between \( n \) and \( z_o \), it is noted that for large values of \( H_w/(ez_o) \), the logarithmic function in equation (5) can be approximated by the classical 1/7 power law \([Chen, 1991; Katul et al., 2002]\)

\[
\ln \left( \frac{H_w}{ez_o} \right) \approx \frac{5}{2} \left( \frac{H_w}{ez_o} \right)^{1/7}. \tag{9}
\]
Combining equations (9) and (5) results in
\[
\frac{U_b}{u^*} = \frac{5}{2\kappa} \left( \frac{H_w}{\varepsilon_{zo}} \right)^{1/7},
\]
(10a)

\[
n = \left( \frac{H_w^{1/6}}{\varepsilon_{zo}} \right) \left( \frac{2 \rho c e^{1/7}}{5 g^{2/7}} \right) z_w^{1/7} \approx 0.06 z_w^{1/7},
\]
(10b)

\[
\sqrt{\frac{f}{8}} = \left( \frac{2 \rho c e^{1/7}}{5} \right) \left( \frac{z_w}{H_w} \right)^{1/7} \approx 0.18 \left( \frac{z_w}{H_w} \right)^{1/7}.
\]
(10c)

Equations (5) and (7) can also be combined to yield \( n = (\kappa/\sqrt{g}) H_w^{1/6} \ln(H_w/z_w) - 1 \) or \( \sqrt{f/8} = (1/\kappa) \ln[H_w/(\varepsilon_{zo})] \) without invoking any power law approximation to the log-profile. The dependence of \( n \) on \( g^{-1/2} \) was pointed out by a number of authors [Chow, 1959; Gioia and Bombardelli, 2002; Yen, 1992], and it was used to adjust tabulated \( n \) values to the gravitational field of Mars in the study of Martian channel formations [Carr, 1979]. The relationship \( n \approx 0.06 z_w^{1/7} \) is also consistent with a number of other studies that employed power law approximations for \( U(z) \) to predict \( U_b \) for hydro-dynamically rough surfaces [Katul et al., 2002]. Recall that for large \( z_w \), \( z_{zo} \), and \( n \) are independent of \( z_w \). Hereafter, the results in equations (10) are referred to as the deep-layer formulation given that \( H_w \gg H_c \) [Chen, 1991; Katul et al., 2002; Yen, 1992, 2002].

2.2. Canopy Effects and the Shallow Layer Formulation (\( 1 < H_c/H_w < 10 \))

For a stationary and planar-homogeneous flow in the absence of a mean vertical velocity, the time and planar-averaged mean momentum equation along the longitudinal direction is given by
\[
0 = \rho g S_0 + \frac{\partial \tau}{\partial z} - \rho L_c \delta_{df}. \tag{11}
\]

where \( \tau = \tau_m + \tau_v - \tau_r \) is the total stress, \( \tau_m \) is the viscous stress, and \( \tau_v = \rho \overline{u'w'} \) is the turbulent stress, where \( \overline{u'w'} \) is the momentum turbulent flux at height \( z \); \( z \) is the height from the channel bottom, and \( \delta_{df} \) is the Heaviside step function given by
\[
\delta_{df} = \begin{cases} 
1 & z/H_c \leq 1 \\
0 & z/H_c > 1.
\end{cases}
\]

In the aquatic vegetation literature, it is common to define the drag force as \( 0.5 (U^2)/L_c \) rather than \( (U^2)/L_c \), which implies that the inferred dimensionless drag coefficient \( C_d \) from such studies should be halved prior to using equation (11). In the derivation of equation (11), a number of approximations were invoked, most of which are discussed elsewhere [Finnigan, 2000; Lopez and Garcia, 2001; Nikora et al., 2001; Raupach and Shaw, 1982; Shimizu and Tsuimoto, 1994; Wilson and Shaw, 1977].

Two simplifications are worth noting: (1) dispersive fluxes, which are formed when spatial correlations exist in the time averaged mean momentum equation that are subsequently spatially averaged within the canopy volume; and (2) finite porosity effects. While the dispersive fluxes appear to be small in dense canopies, at least compared to \( u'w' \), they are significant in sparse canopies [Cheng and Castro, 2002; Poggi et al., 2004a; Poggi and Katul, 2008a, 2008b]. We are mindful that their parameterization remains in its infancy and no attempts have been made here to include them. Equation (11) also neglects any finite porosity effects arising from spatial averaging, though such finite porosity effects can be corrected for. In essence, the drag that the canopy elements exert on the fluid leads to a deceleration of the fluid only within the fraction of the volume occupied by the fluid, which is \( 1 - \eta_p \), where \( \eta_p \) is the proportion of volume occupied by canopy elements. Upon volume averaging the concentrated drag force induced by the foliage within \( 1 - \eta_p \), the bulk volume-averaged drag force should be divided by \( 1 - \eta_p \) and is equivalent to reducing \( L_c \) by \( 1 - \eta_p \). If \( \eta_p \ll 1 \), finite porosity effects can be ignored. Alternatively they may be absorbed in \( L_c \) using a reduction factor of \( 1 - \eta_p \), leaving the form of equation (11) unchanged.

The depth-averaged mean momentum balance is given as
\[
0 = \int_0^{H_w} g S \, dz + \frac{\tau(H_w)}{\rho} - \frac{\tau(0)}{\rho} - \int_0^{H_w} \left[ \overline{U(z)^2} \right] \, dz \tag{12}
\]

where, after neglecting the molecular stresses relative to turbulent stresses, results in \( \tau(z) \approx \tau(z) = \overline{u'w'}(z) \). In the absence of any wind-induced or rain-induced shear stresses on the free surface, \( \overline{u'w'}(H_w) \approx 0 \), and for a dense canopy,
\[
\frac{-\overline{u'w'}(0)}{\int_0^{H_w} \overline{U(z)^2} \, dz} \ll 1
\]
so that the force balance reduces to the interplay between the canopy drag force and the weight of the fluid given as
\[
\int_0^{H_w} \overline{U(z)^2} \, dz \approx g S_0 (H_w - H_c). \tag{13}
\]

Hypothetically, if the mean velocity profile is uniform with \( \overline{U(z)} = U_b \) and \( L_c \) is independent of \( z \), then equation (13) leads to \( (H_w/L_c)[U_b]^2 \approx u^2 \) and \( 8/f = U_b^2 / u^2 \approx H_w/L_c \) [Poggi et al., 2009]. This analysis demonstrates that \( H_w/L_c \) must be one of the dimensionless quantities to be retained in any flow resistance formulation.

Since \( \overline{U(z)} \) is not uniform, a number of approaches can be used to model \( \overline{U(z)} \) within and immediately above canopies, including first-order and higher-order closure models [Baptist et al., 2007; Defina and Bixio, 2005; Neary, 2003; Poggi et al., 2009]. In the work of Poggi et al. [2009], it was shown that a first-order closure model with an imposed mixing length \( (l_{df}) \) that remains constant inside the canopy, hereafter referred to as the canopy layer or
CL but varies linearly as $\kappa(z-d)$ above the canopy, hereafter referred to as surface layer or SL, reproduced measured $\bar{U}(z)$ and $\bar{w}'w'$ profiles from a wide range of experiments reasonably well. Figure 1b shows the assumed shape of $l_{\text{eff}}$ in CL and SL. When variations in $L_{c}$ with $z$ are small, semianalytical models can also be used to estimate $\bar{U}(z)$. One such model can be expressed as follows [Finnigan and Belcher, 2004; Harman and Finnigan, 2007; Massman, 1997; Massman and Weil, 1999; Poggi et al., 2008]:

$$U(z) = \begin{cases} 
U_{\text{SL}}(z) = \frac{u_{*}}{\kappa} \ln \left( \frac{z-d}{z_{o}} \right); & \frac{z}{H_{c}} > 1; \\
U_{\text{CL}}(z) = \frac{u_{*}}{\beta} \exp \left[ \frac{\left( z-H_{c} \right)}{l_{\text{eff}}} \right]; & \frac{z}{H_{c}} < 1
\end{cases}$$

(14)

where $l_{\text{eff}} = 2\beta^{3}L_{c}$ is the constant mixing length inside the canopy and $\beta = u_{*}/U(H_{c})$ is a momentum absorption coefficient. The continuity (i.e., $U_{\text{SL}}(H_{c}) = U_{\text{CL}}(H_{c})$) and smoothness $\frac{dU_{\text{SL}}}{dz} \bigg|_{z=H_{c}} = \frac{dU_{\text{CL}}}{dz} \bigg|_{z=H_{c}}$ (i.e., ensuring a continuous $\bar{w}'w'$) of the mean velocity profile at $z/H_{c} = 1$ necessitates unique relationships between $z_{o}$ and $d$ as a function of $\beta$ and $L_{c}/H_{c}$ given by

$$\frac{d}{H_{c}} = 1 - \frac{2\beta^{3}L_{c}}{\kappa H_{c}}$$

$$\frac{z_{o}}{H_{c}} = \left( 1 - \frac{d}{H_{c}} \right) \exp(-\kappa/\beta) = \frac{2\beta^{3}L_{c}}{\kappa H_{c}} \exp(-\kappa/\beta).$$

(15)

[20] A two-layer (i.e., $z/H_{c} > 1$ and $z \leq H_{c}$) representation for the mean flow field has been used by a number of authors to arrive at approximate flow resistance formulations [Huthoff et al., 2007; Yang and Choi, 2010]. In deriving the effective flow resistance, these studies primarily focused on determining the depth-averaged velocity in each of the two layers without using any smoothness conditions imposed on $\bar{U}(z)$ as done in equations (14) and (15). Also, it should be emphasized that the mixing length is continuous but not smooth thereby resulting in a continuous and nonsmooth turbulent viscosity at $z/H_{c} = 1$.

[21] The near-exponential mean velocity profile shape inside the canopy and the near-logarithmic mean velocity profile shape above the canopy are supported by atmospheric boundary layer (ABL) and flume experiments for dense canopies in which $1 < H_{c}/H_{L} < 5$ (Figure 1c). Moreover, they represent well zones often used to characterize the mean flow through submerged vegetation [Baptist et al., 2007] except immediately close to the ground and the bottom layers of the canopy. In the case of ABL experiments, the leaf area density profiles, especially for the forested ecosystems in Figure 1c, are far from vertically uniform, and yet the canonical mean velocity profile shapes remain near-exponential inside the canopy and near-logarithmic above. For the aquatic vegetation, departures from the exponential form in the CL can occur in the mid to bottom layers of the canopy ($z/H_{c} < 0.4$), where $a$ is generally low [Nepf and Vivoni, 2000]. This is also the case for the hardwood canopy for the ABL experiment of Figure 1c. These secondary peaks are often connected with the presence of a mean pressure gradient or large vertical gradients in the flux-transport terms [Katul and Albertson, 1998; Katul and Chang, 1999; Shaw, 1977; Wilson and Shaw, 1977]. However, much of the bulk velocity within the aquatic vegetation remains dominated by the higher $\bar{U}(z)$ in the upper canopy layers, where the exponential mean velocity profile appears to be reasonable (Figure 1c). Furthermore, the mean velocity profile immediately above the canopy appears to follow a logarithmic behavior even though $H_{c}/H_{L}$ is not too large. With this formulation for $\bar{U}(z)$, the normalized bulk velocity is given by

$$\frac{U_{b}}{U_{*}} = \frac{H_{c}}{H_{w} - H_{c}} \left[ \frac{H_{w} - H_{c}}{U_{\text{CL}}(H_{c})} \right] \left[ \frac{H_{w} - H_{c}}{U_{\text{SL}}(H_{c})} \right],$$

(16)

where, as before, $U_{*} = \sqrt{gS_{o}(H_{w} - H_{c})}$ and

$$\frac{U_{\text{CL}}}{U_{*}} = \frac{1}{u_{*}H_{c}} \int_{0}^{H_{c}} U_{\text{CL}}(z)dz = 2\beta^{3}L_{c} \left[ 1 - \exp \left( -\frac{1}{2\beta^{3}L_{c}} \right) \right];$$

$$\frac{U_{\text{SL}}}{U_{*}} = \frac{1}{u_{*}(H_{w} - H_{c})} \int_{H_{c}}^{H_{w}} U_{\text{SL}}(z)dz = \frac{1}{\kappa} \left\{ -1 + \ln \left[ \frac{H_{w} - d}{z_{o}} \left( \frac{H_{w} - d}{H_{c} - d} \right) \left( \frac{H_{w} - d}{H_{c} - d} \right)^{-1} \right] \right\}.$$
Hereafter, the formulation in equation (18) is referred to as the shallow-layer formulation. It asymptotically converges to the deep layer formulation when \( H_c \gg H_w \) and \( H_w \gg \beta_{c_o} \). Equation (18) makes explicit the direct effects of the canopy on \( U_h \) in the CL and their modulating effect on the log-profile in SL. The momentum absorption coefficient \( \beta \) in equation (18) is known to vary with the canopy density for small \( a \) but saturates at about 0.33 for dense canopies as discussed elsewhere [Katul et al., 1998; Massman, 1997; Massman and Weil, 1999; Poggi et al., 2004b; Poggi and Katul, 2008a; Raupach, 1994]. Figure 1c also shows that \( U(H_c)/u_* \) is about 3 for the ABL experiments, which implies that \( \beta = u_*/U(H_c) \approx 1/3 \), a constant for these dense-canopy experiments. For sparser canopies, \( \beta \) may be derived from flume experiments on rods with various densities [Poggi et al., 2004b]. These experiments suggest that for a fixed canopy height, \( \beta = \text{min}(0.135\sqrt{a}, 0.33) \), which is the relationship used here for the aquatic vegetation. However, the formulation by Massman and Weil [1999] can also be used if the effect of vegetation sheltering is known, and is given by

\[
\beta \approx c_1 - c_2 \exp(c_3 C_d LAI/P_m); \quad P_m \approx 1 + c_4 LAI,
\]

where \( c_1 = 0.33 \), \( c_2 = 0.264 \), \( c_3 = 15.1 \), and \( c_4 \in [0, 0.4] \), and \( P_m \) is a sheltering coefficient (\( c_4 = 0 \) when no sheltering corrections are employed). The specification of \( \beta \) is fundamentally connected to the penetration depth (\( \delta_o \)), or the depth referenced from the canopy top at which 90% of the momentum flux (= \( \overline{u'w'} \)) is extracted by the canopy [Nepf and Vivoni, 2000]. The penetration depth is often used to categorize aquatic vegetation into sparse (\( \delta_o/H_c = 1 \)) and dense (\( \delta_o/H_c < 1 \)) [Nepf et al., 2007]. Using the mean velocity profile in equation (14) and a first-order closure approximation for \( \overline{u'w'}/u_*^2 = -\overline{P_{eff}}(dU_{CL}/dz)^2 \), the momentum flux inside the canopy is given by

\[
\overline{u'w'}/u_*^2 = -\exp\left(\frac{z - H_c}{\beta H_c}\right).
\]

Upon setting \( z = \delta_o, \overline{u'w'}/u_*^2 = -0.1 \), and solving for \( \delta_o \) results in a linear dependence between \( \delta_o/H_c \) and \( (C_d LAI)^{-1} \) given as

\[
\delta_o/H_c \approx -\ln(0.1)/\beta H_c \approx 2.3\beta^2/C_d LAI
\]

when \( \beta \) is constant. The linear relationship between \( \delta_o/H_c \) and \( (C_d LAI)^{-1} \) has also been reported across a wide range of experiments for aquatic vegetation [Nepf and Ghisalberti, 2008]. In airflow within terrestrial vegetation, LAI must exceed 2.3\( \beta^2/C_d \) for the “dense” canopy criterion to be met, i.e., \( \delta_o/H_c \leq 1 \). If \( \beta \approx 0.3 \) (see ABL experiments in Figure 1c) and for a typical \( C_d \approx 0.2 \), this result suggests that “dense” canopies must have \( \text{LAI} \geq 1 \text{ m}^2 \text{ m}^{-2} \), a reasonable choice given that the data sets for the ABL in Figure 1c all have \( \text{LAI} \geq 3 \text{ m}^2 \text{ m}^{-2} \). Moreover, for \( \text{LAI} > 3.5 \) (typical of forested ecosystems), equation (21) suggests that almost 90% of the momentum is extracted in the top 30% of the canopy, which is consistent with empirical findings for a number of atmospheric flow experiments through dense canopies [Katul et al., 2004].

3. Data Sets

The data sets used to evaluate the model in equation (18) spans a wide range of canopy types, \( H_w, H_c, \) and \( a \) as summarized in Table 1 of Poggi et al. [2009]. They include wooden dowels, stainless steel rods, plastic plants, and real plants. For rigid canopies, 53 data sets were used and are described elsewhere [Ghisalberti and Nepf, 2004; Lopez and Garcia, 2001; Meijer and Van Velzen, 1999; Murphy et al., 2007; Poggi et al., 2004b].

For the flexible vegetation datasets, a variety of experiments were also employed that include data sets on wheat stems, mixed grasses, Spartina anglica, plastic aquarium plants, and natural reeds [Baptist, 2003; Carollo et al., 2002; Ciraulo and Ferreri, 2007; Jarvela, 2005; Kouwen and Unny, 1969; Meijer and Van Velzen, 1999; Nepf and Vivoni, 2000; Shi et al., 1995], and one data set from a plastic plant prototype [Nepf and Vivoni, 2000]. In these cases, the reported deflected height rather than the geometric height of the vegetation was used as \( H_c \). When comparing model calculations with the experiments, measured \( S_o, H_w, H_c, a \) and an estimate of \( C_d \) are needed. These variables are presented in Table 1 of Poggi et al. [2009] and are not repeated here. The \( \beta = \text{min}(0.135\sqrt{a}, 0.33) \) and \( L_c = (C_d a) \) can be directly computed from such measurements. The values of \( z_o \) and \( d \) are then determined from equation (15), and \( U_h/u_* \) and \( f \) are computed from equation (18).

4. Results

As earlier noted, in studies that utilize \( U(z) \), the surface roughness is generally characterized by \( z_o \) while in studies that utilize \( U_h \) (as may be the case in flood routing), \( f \) or \( n \) are often preferred. To explore biases arising from using the deep layer formulation when linking these two surface roughness measures, the following analysis was conducted. The value of \( z_o \) was determined from equation (15) using inferred \( L_c, H_c, \) and \( \beta \) for all the data sets. Then, equation (10c) was used to estimate \( f \). The computed and measured \( f \) values are compared in Figure 2. Figure 2 shows that the deep-layer formulation recovers the approximate 1/7 power law evidenced by the overall patterns in the combined data set, but perhaps most evident in the rod canopy experiments [Poggi et al., 2004b]. This rod canopy data [Poggi et al., 2004b] was collected at \( H_w/H_c = 5 \), and hence, the experiment is transitioning toward a deep layer behavior. The mixed grasses experiment [Carollo et al., 2002], which includes the flexible canopy runs with most bending (and possibly reduced drag) appears to have a reduced friction factor when compared to the deep-layer formulation predictions. This data set is the only data set with measured \( f \) smaller than predictions from equation (10c). When comparing the deep layer formulation predictions with the predictions from the shallow layer formulations, it is evident from Figure 2 that the shallow layer formulation always yields a larger \( f \) than its deep layer counterpart. In Figure 2, the deep-layer expression \( \sqrt{f/8} = (1/\kappa)\ln[H_w/(\varepsilon z_o)] \) is also presented for
A comparison between predicted (dashed and dotted lines) and measured $\sqrt{f/8}$ (symbols) as a function of the dimensionless momentum roughness length ($z_w/H_w$), where $z_w$ is computed from equation (15). Predictions (dashed and dotted lines) are based on the deep-layer formulations for a power law and a log-law mean velocity profile. Note that the deep-layer formulations (dashed and dotted lines) underestimate the majority of the experiments. The prediction from the shallow-layer formulation (equation 18) is also presented as blue open circles.

While these analytical model-data comparisons are encouraging, a number of issues remain problematic and need to be confronted. First, the analysis here focused on rigid canopies, and flexible canopies were treated as rigid with a modified canopy height set to the specified bending height. The immediate consequences of this approximation are that the model suppresses any interactions between the bulk flow and vegetation bending. Correcting for such an interaction is difficult within large-scale hydrologic modeling as the stiffness and bending properties of the vegetation and its multiscale architecture become essential. Second, the closure model primarily dealt with submerged canopies, though the flow through emergent canopies can also play a role in flood routing, especially for overbank flow problems and flow in wetlands. Third, first-order closure models were employed throughout and any nonlocal contributions to momentum transfer originating from the flux-transport term were neglected. Likewise, the dispersive stresses in the volume-averaged momentum balance were ignored. Fourth, the adjustment length scale was assumed to be constant independent of $z$, which is clearly not realistic for complex canopy morphology. Fifth, the mean momentum balance treatment was assumed to be entirely one-dimensional (vertical), though in typical flood routing cases, the advective and nonsteady terms inside the canopy can be significant and contribute to the force imbalance in equation (12). Sixth, $\overline{f^t w}$ ($H_w$) was neglected, which is not likely to hold during rain or strong wind events. Despite all assumptions and limitations discussed above, equation (18) does provide a physically based relationship between $n$ or $f$ and $H_w$ under...
a set of restrictive conditions. The significance of this dependence and its consequence on flood routing mechanics are discussed next.

5. Application to Flood Routing

The basic equations that describe the one-dimensional flood routing mechanics along the longitudinal direction ($x$) are the continuity and Saint-Venant’s equations given by

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0
\]

\[
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + gA \left( \frac{\partial H_c}{\partial x} - S_o \right) + gAS_f = 0
\]

where $t$ is time, $Q$ is the total flow rate, and $A$ is the cross-sectional area.

\[
S_f \approx \left( \frac{n Q}{AR_{ch}^{2/3}} \right)^2
\]

The two formulations make the same assumptions that $u_s = \sqrt{g(H_w - H_c)S_o}$. In essence, for operational flood routing approaches employed for vegetated surfaces, the effects of the canopy elements on $n$ are replaced by a hydro-dynamically equivalent $n$ generated from the wetted

Figure 3. Comparison between measured and modeled $n_f$ and $U_b$ for all the experiments using the shallow layer formulations. Symbols are the same as in Figure 2. The one-to-one line (dashed) is also shown. The relative error $e_R = \Delta U_b/U_b$ as a function of the dimensionless adjustment length scale $H_c/L_c$, the relative submergence depth $H_w/H_c$ and the canopy Reynolds number $Re_c = u_cH_c/\nu$ are also presented. A ±20%relative error bounds (horizontal lines) are shown for clarity.
perimeter and responds to variations in $H_w$ as in equation (22b). This is precisely how the effective $n$ was defined and determined in equation (18). As earlier noted, operational flood routing models assume $n$ to be a constant independent of $H_w$. [HEC-RAS, 2002]. However, coupling the friction factor with $H_w$ to generate what can be termed as stage-dependent friction factor has been successfully employed in surface water routing [Casas et al., 2010]. To explore how such nonlinear coupling between $n$ and $H_w$ affects the flood routing mechanics, two separate analyses are conducted and discussed. The first is a scaling analysis on a similarity solution to equation (22), and the second is centered on a numerical analysis of the full Saint-Venant equations with two sets of model calculations contrasted: one with $n$ maintained as a constant in $x$ and $t$, while another evolves $n$ in $x$ and $t$ based on the dynamics of $H_w$ given by equation (18). Hereafter, scenarios with constant and evolving $n$ are referred to as “static” and “dynamic,” respectively.

### 5.1. Scaling Analysis

[30] The scaling analysis here explores how perturbations introduced by the presence of vegetation in the exponent describing the rating curve of $Q - H_w$ impacts the space-time evolution of $H_w$. For illustration, a rectangular channel section whose width $B$ is much larger than $H_w$ is considered. Moreover, the mean momentum balance in equation (22) is considerably simplified by assuming that $S_T \approx S_o$ (i.e., locally steady and uniform). Under these assumptions, $Q$ and the flow rate per unit width $q$ are related by $Q = B q = B (V H_w) = B a_1 (H_w)^m$, where for the static cases $a_1 = (S_o^{1/2}/n)$ and $m = 5/3$ so that the mean continuity equation reduces to

$$\frac{\partial H_w(x,t)}{\partial t} + \frac{\partial}{\partial x}[a_1 H_w(x,t)^m] = 0. \quad (23)$$

[31] In the static case, $n \approx n_o$ and is assumed constant, while in the dynamic case, it is approximated by $n \approx n_o (H_w/H_w^c)^{\omega}$ so that when $\omega \approx 0$ the static case is recovered. This approximation is not exact but captures the canonical shape of equation (18). Hence, a finite $\omega$ leads to an $a_1 = (S_o^{1/2}/n_o)/H_w^c$ and an $m = 5/3 + \omega$. By propagating the effects of a finite $\omega$ on $a_1$ and $m$, and subsequently on the solution to equation (23), a description of how a dynamic $n$ modifies flood routing can be explored without explicitly solving equation (23). It should be noted here that for $\omega = 1/3$ and $m = 2$, equation (23) reduces to an inviscid Burger equation known to admit similarity solutions and shockwaves [Barenblatt, 2003]. When $H_w(x,t) > 0$ for $(x,t) > 0$ and $m \neq 1$, one plausible solution can be derived using the scaling arguments in Appendix A and is given as

$$H_w(x,t) \sim \left(\frac{a_1 m t}{x}\right)^{1/(1-m)} = \left(\frac{n_o}{S_o^{1/2}} \frac{H_w^c}{S_o^{1/2}/n_o} x\right)^{1/(2/(3+\omega)), \quad (24)}$$

[32] Hence, a finite $\omega$ modifies both the exponent and the amplitude of the speed of flood propagation that scales as $x/t$. A formal analysis that includes the effects of a dynamic inflow hydrograph is discussed using numerical analysis, where all the terms in the Saint-Venant equations are retained.

### 5.2. Case Study: Flood Routing Over Grass Swales

[33] For illustrating the effects of a dynamic $n$ on flood routing, a grass swale configuration is employed as a case-study. The setup is not intended to replicate a particular system but resembles designs employed in grass swales along major highways [ISU, 2008]. In grass swale designs, the $S_o$ should be steep enough to ensure adequate velocity, but usually not steeper than 4% to reduce the possible occurrence of a hydraulic jump, with 1% being a recommended design value. Moreover, for storms having a 10-year return period, the expected design $U_k$ should not exceed 1 to 2 m s$^{-1}$, and to enhance particle removal efficiency, a minimum of 10 min residence time for water is recommended. These two constraints imply that the swale length should be on the order of few hundred meters to about kilometer. For the model runs here, a length $L_e = 1000$ m and a bed slope of $S_o = 1.0\%$ are selected as typical values. However, unlike the parabolic cross-sectional area used in grass swale design, a rectangular cross sectional area with a constant width $B = 2.5$ m is employed throughout to simplify the depth-area relationship and minimize parameter specifications. This $B$ is commensurate with a recommended bottom width that varies from a minimum of 0.6 m to a maximum of 2.6 m.

[34] Optimal grass swale designs recommend that the grass be regularly mowed to maintain $H_w$ between 0.10 and 0.15 m. Hence, the 1 km channel is assumed to be covered uniformly with rigid vegetation having an $H_w = 0.15$ m, an $L_w = 0.5$ m$^2$, and a $C_d = 0.5$. These canopy characteristics closely follow values reported for short mowed grass [Novick et al., 2004; Thompson and Daniels, 2010], though the grass canopy is not typically rigid as assumed here. Interestingly, the $z_o$ predicted by equation (15) using these assumed $H_w$, $C_d$, and $L_w$ is 0.009 m, which maintains a hydrodynamically rough surface because $z_o > 2z_w$.

Employing the deep layer formulation ($n = 0.062 x_{in}^{1/7}$) to such a $z_o$ value yields an $n \approx 0.031$. This value of $n$ is in excellent agreement with the minimum $n = 0.03$ recommended in grass swale designs when the flow depth is expected to exceed 0.3 m (i.e., $H_w/H_w^c \geq 2$) as discussed elsewhere [ISU, 2008]. For $H_w/H_w^c \rightarrow 1$, the shallow layer formulation in equation (18) predicts an $n \approx 0.14$, which is also in good agreement with the maximum recommended design value for $n \approx 0.15$ when the water level at the design flow rate are comparable to or smaller than $H_w$ [ISU, 2008]. Moreover, this $n$ is in good agreement with the measured value reported for short grass prairie [Engman, 1986]. Hence, as logical bounds to the static $n$ model runs, these two extreme $n$ values are employed. For initial conditions, it is assumed that the flow is uniform with $H_w/H_w^c = 1.0$ in all model runs.

[35] Boundary conditions are given by an idealized inflow hydrograph approximated by

$$Q_w(0,t) = Q_o \exp \left[-\Theta_1(t - \Theta_2)^2\right] + Q_{di}, \quad (25)$$

where $Q_o$ is the maximum amplitude, $\Theta_1$ and $\Theta_2$ are parameters describing the spread and time to peak of the inflow hydrograph, respectively. The model parameters used in this setup are listed in Table 1.
The modeled hydrographs \( Q(x,t) \) and water depth \( H_w(x,t) \) are presented, respectively, in Figures 4 and 5 for specified locations along the channel. The dynamic \( n \) values at the same spatial locations are also shown in Figure 6. The model results are conducted for a \( Q_o = 4 \text{ m}^3/\text{s} \). These results suggest that the flow rate and concomitant water depth differences between the dynamic and the static run employing the minimum \( n \) are, to a first approximation, minor across the various stream locations. The opposite is true when comparing the dynamic \( n \) with the static run employing the maximum \( n \). For this large \( Q_o \), the dynamic \( n \) rapidly drops to values comparable to the minimum static \( n \) as the flood wave advances (Figure 6), resulting in flood routing mechanics resembling a static case with a friction factor given by the deep layer formulation. The flood propagation velocity and maximum \( H_w \) are comparable for

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Attributes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td>Channel width (m)</td>
<td>2.5</td>
</tr>
<tr>
<td>( L_x )</td>
<td>Channel length (m)</td>
<td>1000</td>
</tr>
<tr>
<td>( S_o )</td>
<td>Bed slope (m m(^{-1}))</td>
<td>0.01</td>
</tr>
<tr>
<td>Vegetation Attributes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAI</td>
<td>Leaf area index (m(^2) m(^{-2}))</td>
<td>0.5</td>
</tr>
<tr>
<td>( C_d )</td>
<td>Drag coefficient</td>
<td>0.5</td>
</tr>
<tr>
<td>( H_c )</td>
<td>Canopy height (m)</td>
<td>0.15</td>
</tr>
<tr>
<td>( n )</td>
<td>Manning’s roughness</td>
<td>Two static cases: 0.03 (deep layer formulation); 0.14 (shallow layer formulation). Dynamic case: See equation (18).</td>
</tr>
<tr>
<td>Initial and Boundary Conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H_w(0,x) )</td>
<td>Initial uniform water level (m)</td>
<td>( H_w )</td>
</tr>
<tr>
<td>( Q_{in} )</td>
<td>Inflow hydrograph (m(^3) s(^{-1}))</td>
<td>( Q(t,0) = Q_o \exp(-\Theta_1(t - \Theta_2)^2) + Q_{o,i} )</td>
</tr>
<tr>
<td>( Q_o )</td>
<td>Amplitude of flood inflow hydrograph, and ( Q_o ) is the ( Q ) at the initial condition ( H_w(0,x) ). ( Q_o \in [2.5, 12.5] ) Unless otherwise stated, ( Q_o = 4 \text{ m}^3/\text{s} )</td>
<td></td>
</tr>
<tr>
<td>( \Theta_1, \Theta_2 )</td>
<td>Spread and time to peak of ( Q_{in} )</td>
<td>(-0.0001 \text{ s}^{-2}, 250 \text{ s} )</td>
</tr>
</tbody>
</table>

Figure 4. The effects of dynamic \( n \) on the hydrographs along various positions across the channel length for the setup in Table 1 and for \( Q_o = 4 \text{ m}^3/\text{s} \). Model calculations with dynamic \( n \), computed using equation (18), are shown in solid lines and the ones computed with a constant \( n \) are shown in dashed lines for the deep layer formulation (minimum \( n \)) and in dots for the shallow layer formulation (maximum \( n \)). The inflow hydrograph (thick-solid) is also shown in the top left plot for reference.
Figure 5. Same as Figure 4 but showing the profile of $H_w$ rather than $Q$.

Figure 6. Same as Figure 4 but showing the space-time evolution of the dynamic Manning’s roughness $n$. The dashed and dotted horizontal lines are for the two “static” $n$ cases derived from deep layer formulation (minimum $n$) and shallow layer formulation for $H_w = 1.01 H_c$ (maximum $n$).
these two end-member cases, at least when compared to the results for the static $n$ inferred from the shallow layer formulation. In essence, these model results lend some theoretical support to the choice of a constant $n$ set to the minimum recommended for the high flood flow rate [ISU, 2008]. Naturally, with such a high $Q_o$, $H_w/H_c$ at the peak of the advancing wave becomes sufficiently large so that $n$ (1) becomes nearly independent of $H_w$ and (2) approaches a near constant value ($= 0.06 z^{1/7}$) as evidenced by Figure 6.

[37] A natural follow-up question then is to what degree this emerging picture is altered for various $Q_o$. To explore this point further, the above model runs were repeated for $Q_o$ that varied from 3 to 13 m$^3$ s$^{-1}$. For each $Q_o$, the maximum flow rate at each $x$ location along the channel was computed for the dynamic and the two static runs. The similarity in these maxima was then assessed using the dimensionless ratios $\Delta Q/Q$, where $\Delta$ is the different between the dynamic solution and one of the two static solutions, normalized by the dynamic solution. A negative dimensionless ratio implies that the static runs over-predict their dynamic counterpart.

[38] For each $Q_o$, the regions along the stream where the dynamic and static predictions of maximum flow rates diverged most are presented in Figure 7. It is clear from Figure 7 that the static solution with $n$ determined from the deep layer formulation notably diverged from the dynamic solution at very low $Q_o$ (<2 m$^3$ s$^{-1}$) and at large $x/L_x$ (>0.7). Otherwise, the dynamic and static solutions for this minimum $n$ differ by no more than 20%, with the static solution over predicting these extreme flow rates. When $Q_o > 7$ m$^3$ s$^{-1}$, the over predictions diminish to less than 20%. For

![Figure 7](image_url)
the static runs at maximum \( n \), the agreement with the dynamic solution is primarily confined to the upstream portion \((x/L_u < 0.2)\) but for all \( Q \). Beyond this range, the differences between this static and dynamic case can exceed 90%.

6. Discussion and Conclusions

[39] The use of dimensional analysis to describe the friction factor over rough surfaces as a function of the Reynolds number is one of the main successes in hydraulic research. Extending the dimensional analysis to canopy flows remains a major scientific challenge because the height and spacing of the vegetation elements can be comparable to the largest eddy and water depth, and the dissipation of turbulent kinetic energy by drag elements can occur over distances that can be comparable to the canopy height rather than the viscous dissipation length scale. Such a setup does not readily admit to a clear scale separation between the eddy sizes transporting momentum to a rough surface (e.g., attached eddies to the boundary) and their interaction with sizes or spacing of the protruding canopy elements [Huthoff et al., 2007].

[40] The case of submerged vegetation flow at sufficiently large Reynolds number was considered so that viscous effects can be neglected. Even for this limiting case, an analysis on the governing terms for the mean momentum balance suggest that the canopy height, the water depth, and the adjustment length scale must be included in any dimensional consideration. Using first-order closure principles, an analytical model for the flow resistance factor was proposed and tested across a wide range of data sets that includes these three length scales. Despite the numerous simplifications made, the agreement between measured and modeled resistance factors and bulk velocities were reasonable to within 20% for several data sets that employed rigid canopies. The model recovers the flow resistance formulation when the water depth is much larger than the canopy height, at least for hydrodynamically rough surfaces. The model also provides a unifying framework that links a number of parameters and length scales often used in canopy turbulence studies such as the penetration depth, the momentum absorption coefficient, the adjustment length scale, the momentum roughness length, and the zero-plane displacement height. While simplifications were made for analytical tractability, the model performance remained comparable to first-order closure model results reported in the literature for the same data sets. The implication on routing mechanics of a friction factor varying with water depth emerging from this theoretical consideration was also explored. Again, commencing with a scaling analysis, it was shown that a water depth dependent friction factor slows down the propagation velocity of an advancing flood wave relative to the deep layer case. Numerical simulations to the full Saint-Venant equations demonstrate that as the flood wave advances, the \( H_u/H_c \) increases locally thereby diminishing the value of \( n \), which speeds up the peak of the flood wave compared to the neighboring fluid velocities. The numerical results show a reasonable match between empirical design standards for grass swales and predictions from the model here.

[41] From a broader perspective, advancements in Interferometric Radar measurements of water level fluctuations from space [Alsdorf et al., 2000, 2001, 2005, 2007a, 2007b; LeFavour and Alsdorf, 2005; Mason et al., 2003; Smith, 1997] and the rapid progress in air-born canopy Lidar measurements [Lefsky et al., 2002a, 2002b; Casas et al., 2010] permit (1) the key canopy attributes (LAI and \( H_u \)) and (2) repeated measurements of water level maps in time to be performed at unprecedented spatial resolution over large basins. Some preliminary attempts using such data for flow rate estimates over large basins are promising [Alsdorf et al., 2007a]. As earlier mentioned, the variables needed to predict the flow resistances here include leaf area index, canopy height, water depth, and estimates of the foliage drag coefficient. While the first three variables can be determined from remote sensing platforms, the approach proposed here is “tempered” by the fourth variable (\( C_d \)). Nonetheless, the proposed approach here is analytical thereby permitting efficient implementation in flood routing across large basins, and specifying \( C_d \) (a foliage attribute) remains constrained (in both space and time).

Appendix A: Validation of the Proposed Similarity Solution

[42] A possible solution to the partial differential equation (pde) given by

\[
\frac{\partial H_u(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[ a_1 H_u(x,t)^m \right]
\]

was proposed for \( m \neq 1 \) and was expressed as

\[
H_u(x,t) \approx \left( \frac{a_1 m t}{x} \right)^{1/(1-m)}.
\]

[43] The genesis of this solution is a scaling analysis on the pde that commences with the argument that

\[
\frac{\delta H_u}{\delta t} \approx a_1 m H_u^{m-1} \frac{\delta H_u}{\delta x},
\]

which can be expressed as

\[
\frac{\delta t}{\delta x} \approx \frac{1}{a_1 m H_u^{m-1}} \sim \frac{t}{x},
\]

resulting in

\[
\left( \frac{1}{H_u^{m-1}} \right)^{1/(1-m)} = H_u \sim \left( \frac{a_1 m t}{x} \right)^{1/(1-m)}.
\]

[44] To verify that this scaling result satisfies the above PDE, it is replaced in the left-hand side (LHS) and the right-hand side (RHS) for an equality check. This replacement leads to the following:

let \( \Psi = \frac{a_1 m t}{x} > 0 \),

\[
\frac{\partial H_u}{\partial t} = \frac{a_1 m (\Psi)^{1-1/[1/(1-m)]}}{(1-m)x};
\]

\[
-\frac{\partial}{\partial x} (a_1 H_u^m) = \frac{(a_1 m)^2 \Gamma(\Psi)^{1-1/[1/(1-m)]} (\Psi^{m-1}/(1-m))}{2 \Gamma(\Psi)^m (1-m)x^2}.
\]
[45] For LHS to be identical to the RHS, the condition

$$\frac{(\alpha m)_{r}}{x} \left(\Psi_{m-1}(1-m)\right) = 1,$$

must be satisfied for all $\Psi$, $x$, and $r > 0$. Provide $m \neq 1$, this condition is always satisfied since

$$\Psi_{m-1}(1-m) = 1.$$

This completes the proof that the proposed similarity solution satisfies the PDE. It must be emphasized that this solution was intended to show how a finite $\omega$ impacts the canonical scaling laws emerging from the PDE rather than any precise matching to specific initial and boundary conditions imposed on the flood routing problem.

[46] Acknowledgments. The authors thank Sally Thompson and Costantino Manes as well as three anonymous referees for all the helpful comments. Support from the Fulbright-Italy distinguished scholars program and the National Science Foundation (NSF-EAR-103359) are acknowledged. Poggi also acknowledges support from the Commission of the European Communities’ WARECALC program (PIRSES-GA-2008-230845).

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