Maximum discharge from snowmelt in a changing climate

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[1] Predicted changes in precipitation and air temperature patterns can lead to major alterations in timing and volume of mountain snowmelt runoff with a possible increased incidence of catastrophic events such as flooding and summer droughts. Here, the role of the temperature seasonal cycle and the relative duration of cold and warm seasons on the partitioning of precipitation into snow and rainfall, snow accumulation and melting dynamics, and the resulting mountain runoff formation are investigated. Using a minimalist analytical model, it is shown that while increased air temperatures reduce snow accumulation in the winter, thus reducing the subsequent snowmelt volumes, they also intensify the rate of snowmelt, thus increasing the discharge peaks per given accumulated snow. The main consequence is the existence of an optimal energy input for which the annual peak discharge reaches an absolute maximum. Such maximum separates a cold regime, where peak discharge is limited by slow melting dynamics, from a warm regime in which peak discharge is reduced by decreased winter snow accumulation. Citation: Molini, A., G. G. Katul, and A. Porporato (2011), Maximum discharge from snowmelt in a changing climate, Geophys. Res. Lett., 38, L05402, doi:10.1029/2010GL046477.

1. Introduction

[2] In many regions of the world, streamflow from snowmelt is a major source of freshwater for human and agricultural use. Such resource is now vulnerable to climatic shifts [see Barnett et al., 2005, and references therein]. In particular, fluctuations in precipitation statistics and increases in mean air temperature can lead to major alterations in timing and volume of runoff, and both these effects may increase the incidence of catastrophic events such as flooding and summer droughts [Day, 2009; Déry and Wood, 2006; Ehsanzadeh and Adamowski, 2010; Hidalgo et al., 2009; Hodgkins and Dudley, 2006; Zappa and Kan, 2007]. This motivates the efforts to clarify the connections between snow accumulation and ablation dynamics, mountain runoff formation, and projected climatic trends [Adam et al., 2009; Jefferson et al., 2008; Stewart et al., 2005].

[3] Because of the complexity of these connections, the problem is usually approached at a local/regional scale by coupling distributed hydrological models – calibrated with data from a wide range of time and space scales – with suitably downscaled Global Circulation Models (GCMs) projections [Bales et al., 2006; Barnett et al., 2005, 2008; Cayan et al., 2008; Day, 2009; Dyer and Mote, 2006; Grundstein, 2003; Huss et al., 2008; Marsh, 1999]. While this approach may be useful at short time scales (daily to seasonal), significant knowledge gaps still persist over longer (seasonal to decadal) time scales. Furthermore, the highly-parameterized and site-specific nature of distributed models, combined with uncertainties associated with GCMs disaggregated projections and short instrument records, makes it difficult to single out the dominant components of snowmelt-generated streamflow dynamics. Such dominant components may be effectively isolated in low-dimensional models. Along these lines, the snow-accumulation and ablation process [Perona et al., 2007; Woody et al., 2009], and flood risk in mountainous areas have been recently explored using simplified stochastic models, and the dynamical systems approach [Allamano et al., 2009; Perona and Burlando, 2008].

[4] In this paper, a minimalist model is proposed that captures the main features of the snow accumulation and melting process and subsequent contribution to streamflow dynamics at the basin scale. The compass of this work is on determining the role of temperature and relative duration of cold versus warm seasons on the partitioning of precipitation into solid and liquid, as well as the accumulation of snow and the intensity of snowmelt. Warm season length is used to represent the integral energy input to the basin that is available for melting. We then show the existence of an absolute maximum in the annual peak discharge as a function of the integral energy input, representing the maximum potential annual discharge that can be produced by a mountain watershed of given storage capacity and average precipitation regime. This maximum peak discharge is a peculiar characteristic of the basin, which separates two types of behavior in the seasonal dynamics of mountain streamflows.

2. Snow Accumulation and Melting

[5] At the annual time-scale, the snow-dynamics over a mountain basin can be described as a storage and release sequence driven by the seasonal dynamics of temperature. For simplicity, we consider the year subdivided into two main seasons: a cold season of duration $T_s$ (snow accumulation season), followed by a warm season of duration $T_w$ (melting season), with $T_{year} = T_s + T_w$. Moreover, the partitioning in solid (snow) and liquid (rainfall) precipitation is assumed to only depend on the alternation between warm and cold seasons – i.e., precipitation falling during the cold season is in the solid form, while rainfall is confined to the warm season. A lumped representation is assumed in space. Accordingly, the precipitation input $r$ is constant and uniformly distributed over the mountainous watershed. Snow transport and redistribution phenomena such as local avalanches, blowing snow and interception by vegetation are neglected, so that snow coverage can be hypothesized to be uniform over the considered drainage basin [De Walle and Rango, 2008; Kind, 1990]. Both the solid precipitation and...
the melting rate are expressed in terms of a snow water equivalent (SWE) depth per unit area and time, without any further hypothesis about the space-time variability of snow density and snow pack metamorphism [Marsh, 1999; Colbeck, 1982]. Melting rate is assumed zero during the cold season and constant during the warm season.

We hypothesize that the daily melting rate \( m \) can be represented as an increasing function of the warm season duration. This is consistent with the fact that the longest warm seasons would also tend to be the warmest ones: if one considers for example a sinusoidal daily average temperature during the year, higher average annual temperatures shift upward the sinusoid and result in longer and warmer summer seasons. For simplicity, the relationship between \( m \) and \( T_w \) is assumed to be linear, i.e.,

\[
m = m_0 \frac{T_w}{T_{\text{year}}},
\]

with \( m_0 \) representing an average maximum annual melting potential that we assume to be about 20–40 mm/day [see De Walle and Rango 2008, chap. 10]. The duration of the warm season \( T_w \) is here a measure of the integral input of energy to the basin so that (1) relates the intensity of the melting to the total energy available for ablation. More complex dependencies do not change the results in a significant way. A linear dependence between the melting rate and the duration of the warm season can affect the correct estimation of the melting phase duration in low-elevated basins with limited accumulation capacity. In this last case, alternative forms of dependence (e.g., power-law) between \( m \) and \( T_w \) may be preferred.

[7] According to the previous assumptions, the time-evolution of the SWE depth \( h(t) \) over a basin is a piece-wise periodic function given by the alternation of a linear accumulation phase with slope \( r \) and a linear ablation phase with slope \( -m \). We consider only cases where a total depletion of the snow mantle takes place before the end of the warm season. This depletion occurs if

\[
rT_w \leq mT_w, \quad (2)
\]

and the duration of the melting phase \( T_m \) can thus be expressed as

\[
T_m = \frac{T_w}{m} (T_{\text{year}} - T_w). \quad (3)
\]

The behavior of \( h(t) \) is sketched in Figure 1 (top), together with the corresponding melting (dashed line) and liquid precipitation (dotted line) intensities.

3. Streamflow Dynamics

[8] With the aim of focusing on the essential linkages between streamflow and snow accumulation-melting dynamics, a simple linear-reservoir model is adopted for the basin response [Bras, 1990; Singh and Singh, 2001]. The introduction of different forms of the response function and non-linear response schemes does not qualitatively affect the results (A. Molini et al., manuscript in preparation, 2011). Under this hypothesis, the annual hydrograph can be obtained as a composition of different seasonal inputs, adding the contributions from rainfall and snowmelt, \( Q_r(t) \) and \( Q_m(t) \), respectively. A schematic representation of the superposition of these two components is also shown in Figure 1.

[9] Adopting a time axis with its origin at the beginning of the warm season, the total (rainfall plus snowmelt) specific discharge during the melting phase, \( 0 \leq t \leq T_m \), is given by

\[
Q(t) = Q_m(t) + Q_r(t) = (m + r) \left( 1 - e^{-\frac{t}{T_m}} \right) + (Q_m(0) + Q_r(0)) \ e^{-\frac{t}{T_m}}, \quad (4)
\]

where \( k \) (expressed in days) is the storage constant of the basin, also representing the characteristic spatial scale of the basin. Especially for large basins, the use of different storage constants maybe more realistic. However, the goal is to isolate the main dynamical processes governing the long-term connections between the total energy input to the basin and the accumulation-ablation process. For this reason, we consider a unique storage constant for the basin, representing its characteristic (and average) storage capacity.

[10] In the remaining part of the warm season \( T_m < t \leq T_w \), the specific discharge due to snowmelt \( Q_m(t) \) starts to recede, while liquid precipitation continues to feed \( Q_r(t) \). Hence, the total specific discharge in this phase of the year is given by

\[
Q(t) = r \left( 1 - e^{-\frac{t}{T_w}} \right) + Q_r(0) \ e^{-\frac{t}{T_w}} + \left[ m \left( 1 - e^{-\frac{t}{T_m}} \right) + Q_m(0) \ e^{-\frac{t}{T_m}} \right] \ e^{-\frac{t}{T_m}}. \quad (5)
\]
with \( K_r = \left[ T_{year} \left( e^{T_{year}/k} - 1 \right) \right]^{-1} \). The seasonal dynamics of \( Q(t) \) over 2 years is shown in Figure 1. Note that the maximum peak discharge \( Q_{max} \) is always located at the end of the melting season \( T_m \) (see Figure 1, bottom), and shifts as a function of \( T_w \). \( Q_{max} \) is then given by

\[
Q_{max} = Q(T_m)
= K_r \left( \psi e^{T_{year}} + e^{T_{max}(T_{year} - T_w)} \right) r T_{year} e^{T_{year}} - \psi e^{T_{year}} - r T_{year} \right).
\tag{10}
\]

being \( \psi = (m_0 T_w + r T_{year}) \). Given its dependence on \( T_w \), \( Q_{max} \) can be used as a measure of the sensitivity of annual peak discharges to climatic variability.

### 4. Maximum Streamflow at Intermediate Temperatures

[12] The previously described annual hydrograph is a function of the duration of the summer season, the storage capacity of the basin \( k \), the precipitation regime \( r \), and the potential melting rate \( m_0 \). Out of these quantities, \( T_w \) is the most important one and can be used to investigate the role of climate change as it represents the integral energy input into the basin and governs the partitioning of precipitation in solid and liquid, as well as the intensity of melting. Longer and warmer summer seasons are expected to result in higher melting rates, but also in a reduction of the snow storage during cold seasons. On the other hand, colder climates allow for an increase in snow accumulation, but also result in slower and less intense melting phases. As a result, when analyzing the annual hydrograph as a function of \( T_w \) (see Figure 2), we observe that the competition between these two trends gives rise to an absolute maximum among annual peak discharges \( Q_{max}^* \), representing the maximum potential discharge that can be produced by a basin of given storage capacity and average precipitation input. This can be obtained by maximizing equation (10) with respect to \( T_w \). The value of \( T_w \) at which this maximum occurs, \( T_w^{max} \), separates two different regimes: a colder one where increasing temperatures produce higher maximum discharge, and a warmer one in which higher temperatures reduce the snow storage during the cold season and thus the subsequent discharge.

[13] This fact is illustrated in Figure 2, showing the annual dynamics discharge as a function of different \( T_w \) for given values of the other parameters \((r = 4 \text{ mm/day}, m_0 = 35 \text{ mm/day} \) and \( k = 15 \text{ days} \). The warm season duration is varied between the minimum \( T_w \) for which condition (2) still holds (blue dashed curve) and the entire year (black dashed curve). It is evident that warm seasons shorter than \( T_w^{max} \), producing less intense melting phases, result in delayed and lower peak flows. On the other hand, higher energy inputs related to \( T_w > T_w^{max} \) anticipate the time to peak and reduce the peak until the discharge becomes uniformly distributed over the year for \( T_w = T_{year} \). It is worth noting that the existence of \( Q_{max}^* \) is the result of the dependence of the melting rate \( m \) on the warm season length, while with \( m \) independent of \( T_w \) no absolute maximum is found, as shown in the inset of Figure 2. Finally, Figure 3 shows the dependence of \( Q_{max}^* \) on the duration of the warm season (and thus the mean annual temperature) for different values of \( k \). Such a dependence is further summa-

![Image](image_url)
Figure 3. Peak discharge $Q_{\text{max}}$ for $r = 4$ mm/day, $m_0 = 35$ mm/day and different values of the storage constant $k$ (ranging between 1 and 30 days) as a function of the warm season duration $T_w$. Inset shows the dependence on $k$ of the warm season duration at which the discharge peak reaches the maximum, $T_w^{\text{max}}$, for different values of the input rates $m_0$ and $r = 4$ mm/day.

5. Conclusions

Despite its simplicity, the minimalist model presented here captures the essential seasonal temperature control on snow accumulation and melting in mountain basins and the subsequent streamflow production. The main result is the possible existence of an optimal energy input for which the annual peak discharge reaches an absolute maximum. Such maximum separates a cold regime, where peak discharge is limited by slow melting dynamics, from a warm regime in which peak discharge is reduced by decreased winter snow accumulation. While a marked dependence of the maximum peak discharges on average annual temperature had been already noticed [see Jefferson et al., 2008, and references therein], the actual presence of this absolute maximum as a function of mean temperature seems to be novel. The existence of this maximum in real basins can be verified by analyzing long-term streamflow data conditioned on specific annual regimes of temperature and precipitation. The model can be made applicable to larger spatial scales by the adoption of a number of revisions including more complex temperature and precipitation seasonal trends and non-linear storage models, as well as different storage components and empirical temperature lapse-rates directly estimated from data (A. Molini et al., manuscript in preparation, 2011). It is also possible that in real basins, the absolute maximum may be masked in part by spatial heterogeneity and temporal (daily, seasonal, and inter-annual) hydro-climatic fluctuations in measured streamflow series.

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References


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