Analytical models for the mean flow inside dense canopies on gentle hilly terrain

D. Poggi, G. G. Katul, J. J. Finnigan and S. E. Belcher

Dipartimento di Idraulica, Trasporti ed Infrastrutture Civili, Politecnico di Torino, Torino, Italy
Nicholas School of the Environment and Earth Sciences, Duke University, Durham, NC, USA
Department of Civil and Environmental Engineering, Pratt School of Engineering, Duke University, Durham, NC, USA
CSIRO Atmospheric Research, FC Pye Laboratory, Black Mountain, Canberra, Australia
Department of Meteorology, University of Reading, Reading, UK

ABSTRACT: Simplifications and scaling arguments employed in analytical models that link topographic variations to mean velocity perturbations within dense canopies are explored using laboratory experiments. Laser Doppler anemometry (LDA) measurements are conducted in a neutrally-stratified boundary-layer flow within a large recirculating flume over a train of gentle hills covered by a dense canopy. The hill and canopy configuration are such that the mean hill slope is small and the hill is narrow in relation to the canopy \((H/L \ll 1\) and \(L_c/L \approx 1\), where \(H\) is the hill height, \(L\) the half-length, and \(L_c\) the canopy adjustment length-scale). The LDA data suggest that the often-criticized linearizations of the advective terms, turbulent-shear-stress gradients and drag force appear reasonable except in the deep layers of the canopy. As predicted by a previous analytical model, the LDA data reveal a recirculation region within the lower canopy on the lee slope. Adjusting the outer-layer pressure perturbations by a virtual ground that accounts for the mean streamline distortions induced by this recirculation zone improves this model’s performance. For the velocity perturbations in the deeper layers of the canopy, a new analytical model, which retains a balance between mean horizontal advection, mean pressure gradient and mean drag force but neglects the turbulent-shear-stress gradient, is developed. The proposed model reproduces the LDA measurements better than the earlier analytical model, which neglected advection but retained the turbulent-shear-stress gradient in the lower layers of the canopy and near the hill top. This finding is consistent with the fact that the earlier model was derived for tall hills in which advection inside the canopy remains small. In essence, the newly-proposed model for the narrow hill studied here assumes that in the deeper layers of the canopy the spatial features of the mean flow perturbations around their background state can be approximated by the inviscid mean-momentum equation. We briefly discuss how to integrate all these findings with recent advances in canopy lidar remote-sensing measurements of general topography and canopy height. Copyright © 2008 Royal Meteorological Society

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1. Introduction

For over 30 years, micrometeorology remained in a paradigm lock with regard to the quantification of exchange processes between the biosphere and atmosphere. Almost all of its theories, measurement techniques and data interpretations are based on flat-world representation. So it comes as no surprise that many reviews on surface–atmosphere mass, energy and momentum exchange have emphasized the urgent need to confront the problem of turbulent flows within plant canopies on non-flat terrain (Finnigan et al., 1990; Milton and Wilson, 1996; Raupach and Finnigan, 1997; Belcher and Hunt, 1998; Finnigan, 2000; Wood, 2000; Baldocchi et al., 2001; Finnigan and Belcher, 2004; Bitsuamlak et al., 2004; Belcher et al., 2007). This need is driven by a wide range of applications, including biogeochemical cycling, surface hydrology, wind engineering and urban canopies, wind energy generation, and numerical weather prediction. Common to all these applications is the recognition that incorporating the simultaneous effects of canopy and topographic variations, even on the most elementary flow variables, is a logical first step to progress on this problem.

Until recently, theoretical progress on this problem has been virtually non-existent, even for idealized cases often synonymous with two-dimensional steady-state and neutral flow within uniform dense canopies on gentle hills. The combination of hill geometry and canopy morphology imposes multiple length-scales, thereby prohibiting unambiguous use of scaling arguments to reduce various terms in the mean longitudinal momentum balance. Notwithstanding this difficulty, Finnigan and Belcher (2004) have proposed an analytical model for the mean flow within the canopy sub-layer (CSL) over hilly terrain for a set of simplifying conditions. This constitutes
the first analytical model that ‘fingerprints’ how gentle topographic perturbations produce variations in the mean velocity within dense canopies. It received immediate attention because of its potential use in computing scalar advection on non-flat terrain (Katul et al., 2006). It is strictly applicable for deep canopies on gentle and long hills, assuming that the pressure in the outer layer is not affected by the flow inside the canopy.

To conceptually clarify some of the assumptions of the Finnigan–Belcher model, Figure 1 presents various flow regimes inside canopies on gentle hilly terrain, with length-scales relevant to topography and canopy attributes: the hill height \( H \), the hill half-length \( L \), the canopy adjustment length-scale \( L_c \) (influenced by the canopy drag and leaf area density), and the canopy height \( H_c \) (see Appendix A for details). For illustration, and assuming a low mean slope \( H/L = 0.1 \), the regimes in Figure 1 classify hills and canopies in a binary manner, as ‘narrow’ \( (L/L_c < 2) \) or ‘long’ \( (L/L_c > 2) \), and as ‘deep’ \( (H_c/L_c > 2\beta^2) \) or ‘shallow’ \( (H_c/L_c < 2\beta^2) \), where \( \beta = 0.3 \) is a dimensionless momentum flux for dense canopies. The envelope shown in Figure 1 \( (H_c/L_c = 2(H/L)\beta^2) \) delineates regions in which the mean vertical velocity inside the canopy is expected to be sufficiently large to significantly affect the outer-layer pressure. This envelope can be derived from a scaling analysis, using the condition that the mean vertical velocity at the canopy top must be sufficiently small so that the outer-layer pressure is unaffected by the presence of the canopy. When this condition is not met, the vertical velocity generated within the canopy can affect the pressure in the outer region (see Appendix A for details).

Hereafter, flow regimes situated above this envelope are referred to as ‘interactive’ pressure regimes, while flow regimes below this envelope are referred to as ‘fixed’ pressure regimes. In addition to gentle slopes, the Finnigan–Belcher model assumes long hills (so that advection remains small), deep canopies (so that the momentum is entirely absorbed by the foliage elements), and fixed pressure regimes (so that the pressure can be predicted from topography and outer-layer scaling). Hence, departures from the model (Regime I) can be attributed to interactive pressure alone (Regime II), advection and interactive pressure (Regime III), or the development of a secondary boundary layer at the ground due to a finite shear stress (Regime IV). Regime V is dynamically rich because all these mechanisms can play a role.

It is clear from Figure 1 that a wide range of parameter regimes are possible in flow over forested hilly terrain, even when the slope of the hill is small. To date, these regimes have not been systematically explored. Ross and Vosper (2005) have used a first-order closure model to numerically explore the roles of some of these parameters, and more recently Poggi and Katul (2007a) have presented laboratory measurements within Regime V. Since interactive pressure and advection are two primary sources of departure from the analysis in Finnigan and Belcher (2004), we comment on them separately.

- The Finnigan–Belcher model predicts the existence of a region of reversed flow within the canopy, close to the ground, on the lee side of the hill, even when \( H/L \ll 1 \). According to Finnigan and Belcher (2004), the increased displacement of the streamlines due to the recirculation zone can affect both the magnitude and the phase of the pressure perturbation. As predicted by Ross and Vosper (2005), the displacement of the streamlines is much stronger for narrow hills than for long ones. A concomitant effect of this streamline divergence is that the pressure perturbation in the outer layer can be significantly different from that assumed by Finnigan and Belcher (2004) (see Figure 1). This point is confirmed by Poggi and Katul (2007a), who show experimentally that in the case of a narrow hill, the asymmetry in the flow-field pattern inside a dense canopy leads to a phase shift in measured pressure perturbations with respect to the terrain surface.

- The model calculations in Ross and Vosper (2005) suggest that advection is negligible for deep canopies and gentle hills, but, as anticipated by Finnigan and Belcher (2004), cannot be neglected for narrow hills (see Figure 1). As pointed out by Ross and Vosper (2005), the assumption that \( L_c/L \ll 1 \) is restrictive in practice: conditions for which \( L_c/L \approx 1 \) remain important, because these are the conditions that tend to promote significant momentum advection inside canopies. Poggi and Katul (2007a) have shown experimentally that advection can be significant for a narrow forested hill in several regions of the CSL inside dense and tall canopies.

With the above discussion in mind, we formulate three interrelated objectives:

- to explore the basic assumptions often used to derive analytical models for flow over gentle hills inside canopies (e.g. linearization of the budget equations and K-theory);
- to explore whether any one of the three major factors illustrated in Figure 1 (ground shear stress, interactive pressure, and advection) is dominant, and if so whether this can help in the development of a quantitative model for the flow within the canopy in Regime V;
- to test the robustness of the Finnigan–Belcher model, and its simplified version used in Katul et al. (2006), by comparing its predictions with flow measurements outside Regime I (for example in Regime V).

To pursue these three objectives, flume experiments are conducted to simulate a neutrally-stratified atmospheric boundary layer over a train of cosine hills covered with a dense plant canopy. The boundary layer is experimentally reproduced in a tilted water channel, and the dense plant canopy is simulated using a regular array of vertical cylinders configured so that \( H/L = 0.1 \) (i.e. gentle hills), \( L_c/L = 1 \) (i.e. narrow hills), and \( H_c/L_c = 0.13 \) (see Figure 1). An important advantage of using a tilted flume with a free water surface is that both the mean pressure...
gradient and the fluctuations produced by topography can be controlled.

Although numerous other factors not accounted for in these experiments – such as density stratification, heterogeneity in canopy morphology, three-dimensional terrain variability with multiple energetic modes, and unsteadiness in the mean flow – are known to be significant in field conditions, accounting for all of them is well beyond the scope of a single experiment. Thus our experiment should be viewed as a starting point from which to explore basic interactions between idealized canopy turbulence and gentle topography within a ‘model complexity’ framework commensurate with that of Finnigan and Belcher (2004) in a regime not previously explored (Regime V in Figure 1).

2. Experimental facilities

The details of the experimental facility are discussed in Poggi and Katul (2007b, 2007c, 2007d), but an overview is provided here for completeness. The velocity and pressure measurements were conducted in the OMTIT recirculating flume (18 m long, 0.90 m wide, and 1 m deep) at the Giorgio Bidone Laboratory at DITIC, Politecnico di Torino, using a flow rate of $Q_r \approx 120$ ls$^{-1}$ (Figure 2).

2.1. Topography and canopy attributes

The hilly topography was constructed using four modules of a wavy stainless-steel wall, each representing a cosine hill with a shape function given by $f(x) = (H/2) \cos(kx + \pi)$, where $x$ is the longitudinal distance, $H = 0.08$ m is the hill height, $k = \pi/2L$ is the hill wavelength, and $L = 0.8$ m (Figure 2). The model canopy was composed of vertical stainless-steel cylinders ($H_c = 0.1$ m and diameter $d_c = 0.004$ m) arranged on the wavy wall with a density of 1000 rods per square metre. The vertical distribution of the rods’ frontal area was designed to resemble the leaf-area density of a tall hardwood forest at maximum leaf-area index: that is, much of the leaf-area density was concentrated in the top third, with a small and almost-constant amount in the bottom two-thirds.
2.2. Coordinate system

The acquisition of the velocity requires that an appropriate coordinate system be defined *a priori*. Although several other coordinate systems are possible (e.g. rectangular Cartesian or terrain-following), we chose a streamline (‘displaced’) coordinate system, which follows the streamlines of potential flow over the hill. This coordinate system reduces to a terrain-following one near the ground and to a rectangular Cartesian one well above the hill, thereby retaining advantages of both in the appropriate regions (Finnigan and Belcher, 2004). The rectangular \((X, Z)\) and displaced \((x, z)\) coordinate systems can be explicitly related by:

\[
\begin{align*}
x &= X + \frac{1}{2} H \sin(kX)e^{-kZ}, \\
z &= Z - \frac{1}{2} H \cos(kX)e^{-kZ},
\end{align*}
\]

where \(H\), \(L\) and \(k\) are schematically shown in Figure 2.

2.3. Velocity measurements

The time series of longitudinal velocity \(u\) and vertical velocity \(w\) were acquired above the third hill module using two-component laser Doppler anemometry (LDA). The LDA measurements were performed at 0.40 m from the lateral wall at 10 longitudinal positions to cover one hill module, and along a large number of vertical positions (about 35) in the displaced coordinate system. The sampling duration and frequency for each run were 300 s and 2500–3000 Hz respectively, and were deemed sufficient to ensure convergence of the flow statistics (Poggi et al., 2002, 2003).

2.4. Pressure measurements

The mean pressure variations along the train of hills were estimated using the hydrostatic assumption. This assumption allows us to compute the mean longitudinal pressure variations from detailed measurements of the water-surface profile. The water surface was measured using a CCD camera mounted on a trolley moving on rails at a speed of 2 m min\(^{-1}\) along the entire length of the flume. High-resolution digital movies (704 × 576 pixels, DV-AVI, PAL format) were acquired at high frequency (25 frames per second) along the four test sections using an interrogation window about 14 cm wide and 12 cm high. To calibrate the camera position with respect to the hill surface, four runs were conducted using still-water conditions. The displacement between the horizontal plane and the
water surface was evaluated by post-processing the frame sequences extracted from these movies. To analyse the phase relationship between the water and the hill surface, a second camera was used to acquire the vertical elevation of the ground simultaneously with the first camera.

3. Models for mean flow inside canopies on gentle hills

Before introducing the analytical models, we briefly review the governing equations and their boundary conditions in the context of this experimental set-up.

3.1. Governing equations

According to Jackson and Hunt (1975), it is convenient to decompose the mean longitudinal momentum balance equation into an unperturbed (or background equilibrium) state and a perturbation induced by topographic variations. With such a decomposition, the effects of topographic variations on the flow dynamics can be readily traced. With this decomposition, the perturbed mean longitudinal momentum balance equation into an unperturbed (or background equilibrium) state and a perturbation induced by topographic variations.

\[
(U_b + \Delta u) \frac{\partial \Delta u}{\partial x} + \Delta w \left( \frac{\partial U_b}{\partial z} + \frac{\partial \Delta u}{\partial z} \right) = -\frac{\partial \Delta p}{\partial x} + \frac{\partial \Delta \tau}{\partial z} - H_d \Delta F_c, \tag{1}
\]

where \( u \) and \( w \) are the temporally (and spatially) averaged velocities, \( \tau \) is the turbulent shear stress, \( p \) is the pressure per unit density, \( F_c \) is the absorption of momentum by the canopy, and \( H_d \) is the Heaviside step function defined by

\[
H_d(z) = \begin{cases} 
1 & z < 0 \\
0 & z > 0 
\end{cases}
\]

The subscript ‘\( b \)’ and the symbol \( \Delta \) indicate the background and the perturbation from the background, respectively.

The magnitude of \( U_b \) here explicitly affects the budget equation of \( \Delta u \); hence the solution for \( \Delta u \) can be sensitive to how \( U_b \) is estimated or modelled (Poggi and Katul, 2007d). Often, above and within the canopy, \( U_b(z) \) is modelled using a combination of a logarithmic and an exponential profile, given by:

\[
U_b(z) = \frac{u_* \log \left( \frac{z + H_c - d}{z_0} \right)}{U_b e^\beta z/L_c} \quad z > 0
\]

\[
= \begin{cases} 
u & z < 0 \\
0 & z > 0 
\end{cases}
\]

where \( u_* \) is the friction velocity at the canopy top, \( d \) and \( z_0 \) are the zero-plane displacement and momentum roughness length of the canopy, respectively, \( k_r = 0.4 \) is von Karman’s constant, \( U_b \) is the mean velocity at the canopy top, \( \beta = u_*/U_b \) is the dimensionless momentum flux through the canopy, and \( L_c \) is a characteristic turbulent mixing length, equal to \( k_r(z + d) \) above the canopy and \( 2\beta^4 L_c = l \) inside the canopy. Note that the set-up here does not allow us to assume a deep canopy \textit{a priori}, because the small value of \( H_c/L_c = 0.13 \) places the canopy conditions within Regime V (see Appendix A). Therefore the common assumption of an exponential mean-velocity profile for the background flow may not be accurate deep in the canopy because of the potential for a secondary boundary layer at the ground. Nevertheless, Poggi and Katul (2007d) have shown that the exponential profile is an acceptable parametrization of the mean background longitudinal velocity for most of the canopy sub-layer. To demonstrate this point, Figure 3(a) compares the exponential profile and the spatially-averaged mean longitudinal velocity (longitudinally averaged along the 10 measured sections). It is clear that the exponential profile represents the data reasonably well. Moreover, Figure 3(b) shows the modelled and measured Reynolds stresses, which unambiguously demonstrate that most of the momentum is absorbed by the canopy elements.

3.2. Parametrization of Reynolds stress, pressure, and drag

3.2.1. Reynolds stress

In order to solve for the mean velocity, we must employ models for \( \tau \), \( p \) and \( F_c \), to ‘close’ Equation (1). For flat terrain, eddy-viscosity closure (hereafter referred to as ‘K-theory’) for \( \tau \) may be reasonable, provided that an appropriate mixing length \( l_{\text{eff}} \) is defined (Poggi et al., 2004a). It remains unclear whether closure models developed over flat terrain can be readily extended to canopy flows on hilly terrain. According to Belcher and Hunt (1998) (see also Hunt et al. (1988)), K-theory may be used to relate the perturbed Reynolds stress to the perturbed local mean-velocity gradient only in regions where the local eddies are in equilibrium with the surrounding flow. If K-theory is adopted for \( \tau \), the perturbed Reynolds stress above the canopy can be expressed as

\[
\Delta \tau = k_r(z + d) \left\{ 2u_* \frac{\partial \Delta u}{\partial z} + k_r(z + d) \left( \frac{\partial \Delta u}{\partial z} \right)^2 \right\},
\]

while the perturbed Reynolds stress within the canopy can be expressed as

\[
\Delta \tau = 4\beta^4 L_c \left\{ U_b(z) \frac{\partial \Delta u}{\partial z} + \beta^2 L_c \left( \frac{\partial \Delta u}{\partial z} \right)^2 \right\}.
\]

In the above derivations of \( \Delta \tau \), the nonlinear contribution from the \( \Delta u \) term is retained for completeness. The importance of this is explored later, using data from the experiment.

3.2.2. Pressure

The main ‘forcing’ on flows inside canopies on complex terrain is \( \Delta p \), produced primarily by topographic...
Figure 3. (a) The normalized mean longitudinal velocity profiles measured at the 10 sections (s1–s10) along with their planar average. The velocity is normalized by the mean inner-layer velocity, and the height is normalized by the inner-layer depth. The standard background velocity formulation is shown in dashed line. (b) The normalized shear stress measured at the 10 sections (s1–s10) along with their planar average. The stresses are normalized by the planar averaged squared friction velocity at the canopy top. (c) Comparison of measured pressure (circles) and modelled pressure (lines) across the hill. The dashed line is the modelled pressure in phase with topography (as in Finnigan and Belcher (2004)); the dash-dotted line is the modelled pressure determined from the virtual ground (shown in panel d). (d) Spatial variation in the LDA-measured longitudinal velocity (ms$^{-1}$) across the hill, along with the 10 sampling sections. The figure shows the hill surface, the mean inner-layer depth, the canopy sub-layer depth (estimated from the inflection point when sweeps and ejections contribute equally to momentum transfer above the canopy), the recirculation zone on the lee side of the hill ($u<0$), and the virtual ground. (e) Mean streamlines determined from the LDA data. All vertical distances are normalized by the inner-layer depth $h_i$; horizontal distances are normalized by the hill half-length $L$. The 10 vertical sections (s1–s10) indicate the LDA sampling locations across the hill. This figure is available in colour online at www.interscience.wiley.com/qj

variations. According to Finnigan and Belcher (2004), $\Delta \rho$ can be considered an ‘external’ forcing imposed by topographic variations on the flow. Both Finnigan and Belcher (2004) and Jackson and Hunt (1975) model the pressure using a characteristic magnitude $\rho_0$ and a dimensionless longitudinal function $\sigma(x)$ representing the leading-order term induced by topographic variations, given by $\Delta \rho(x) = \rho_0 \sigma(x)$, where $\rho_0 = U_0^2$, $U_0$ being a characteristic velocity representing the forcing due to the mean flow field in the outer region. For a two-dimensional sinusoidal hill, $\sigma(x) = -(H/2)k \cos(kx)$ and $\Delta \tilde{\rho}(x) = -U_0^2(H/2)k \cos(kx)$. 

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The pressure perturbation for hills with a general shape can be evaluated using linear superposition (to a leading order). Once the $n$ most representative modes of a generic hill-shape function have been evaluated, the pressure perturbation becomes:

$$\Delta \tilde{p}(x) = -\frac{1}{2} U_0^2 \sum_{i=0}^{n-1} k_i H_i \cos(k_i x),$$

where $H_i/2$ and $k_i$ are the amplitude and the wave number, respectively, of the $i$th mode.

This linear treatment (Finnigan and Belcher, 2004) is based on the assumption that the perturbed pressure is in phase with the topographic variations, which is appropriate for Regime I. Poggi and Katul (2007a) and Ross and Vosper (2005) have shown that this assumption does not carry through to dense canopies on narrow hills, as suggested by the regimes shown in Figure 1. In fact, the data in Poggi and Katul (2007a) show that the asymmetry in the flow pattern inside a dense canopy leads to a phase shift in the measured pressure perturbation relative to its predicted behaviour from topography only. Both Poggi and Katul (2007a) and Ross and Vosper (2005) show that the minimum pressure appears to be shifted downhill from the summit relative to predictions from the above linear analysis. Figure 3 shows the measured (Poggi and Katul, 2007a) and predicted pressures from the three most energetic modes representing a ‘virtual’ ground. Here, a virtual ground has been determined so that it coincides with the real topography on the upstream side of the hill but becomes ‘displaced’ from the real ground by the thickness of the recirculation zone on the lee side of the hill.

### 3.2.3. Drag

For $F_c$, it is assumed that the Reynolds number is sufficiently large so that the viscous drag is negligible compared to the form drag, and:

$$\Delta F_c = C_d a(2U_b |\Delta u| + |\Delta u| |\Delta u|),$$

where $C_d$ is the drag coefficient and $a$ is the leaf-area density. In dense canopies, this approximation for $\Delta F_c$ can break down in regions where $u$ is small. This is discussed later as part of the data-model analysis.

### 3.3. Linearized equations

There are several theoretical and practical reasons to explore the linearized versions of the perturbed equations. When dealing with complex terrain data (such as those obtained from digital elevation maps, invariably possessing multiple Fourier modes), it is often desirable to quantify how each topographic mode ‘resonates’ with the total perturbation in the mean velocity. Linearizing the equations allows us to compute the total perturbation in the mean velocity as the sum of the perturbations induced by the individual topographically-energetic modes; and it also allows us to derive simplified analytical solutions for the mean velocity fluctuations for simple topography (for example, with one mode of variability) and then to extend these via superposition to more complex terrain.

For $H/L \ll 1$, Jackson and Hunt (1975) and Finnigan and Belcher (2004) argue that the hill perturbations are small compared to the background terms above and within the canopy, so that Equation (1) can be linearized as:

$$U_b(z) \frac{\partial \Delta u}{\partial x} + \Delta w \frac{\partial U_b(z)}{\partial z} =$$

$$- \frac{\partial \Delta p(x)}{\partial x} + 2r_{\text{eff}}^2 \frac{\partial^2 U_b}{\partial z^2} \frac{\partial \Delta u}{\partial z} - \frac{H_s}{L_c} \frac{2U_b(z) \Delta u}{L_c}.$$

While such a linearization may be reasonable for gentle hills in the absence of a canopy, it remains questionable for flows inside dense canopies, because $U_b$ can be small and comparable with the perturbation (Finnigan and Belcher, 2004). This linearization remains the cornerstone of all analytical derivations, and is often cited as the main criticism against analytical theories (Wood, 2000).

The dataset from this experiment is used to explore this particular point next.

The Finnigan–Belcher model linearizes three terms in the mean longitudinal momentum balance: the advective term, the drag force in the upper canopy layer, and the shear-stress perturbation. Figure 4 compares the linearized and nonlinear terms using the LDA measurements collected within the canopy and in the inner layer.

#### 3.3.1. Advection

Overall, it is clear from Figure 4(a) that the often-criticized linearization of the advective terms appears to be a reasonable approximation for this set-up, and is not likely to lead to any primary source of error in the modelled $\Delta u$. When only the lower-canopy dataset ($z/H_s < -0.5$) is considered, the magnitude of the sum of the measured advective terms remains large (about 50% of the maximum), and the sum of the linearized advective terms underestimates the measured sum by more than 50%. This is the first quantitative suggestion from the data that advection within the canopy is indeed important, as suggested by Figure 1.

#### 3.3.2. Drag force

Similarly, the linearized drag force appears to represent the overall drag force reasonably well (Figure 4(b)). However, in the lower canopy layers, where $\Delta u$ is large and $U_b$ is small, the linearization significantly underestimates the drag force (confirming earlier comments by Finnigan and Belcher (2004)).

#### 3.3.3. Shear stress

The linearized shear stress appears to match the nonlinear form well in the inner layer and the upper canopy layer.
Figure 4. Global comparison between the linearized and nonlinear terms in the perturbation budget (ms$^{-2}$), for: (a) advective acceleration; (b) drag force; (c) turbulent shear stress. The closed symbols represent the lower canopy layers only (defined by $z/H_c < -0.5$). The values of $U = U_b + \Delta u$, $U_b$, $\Delta u$ and $\Delta w$ are all computed, along with their gradients, from the LDA measurements.

(Figure 4(c)). However, in the lower canopy layer, it is clear that it significantly underestimates the magnitude of its nonlinear counterpart (by a factor of 4). In these regions, the Finnigan–Belcher model assumes that the shear stress is small in relation to the other terms in the mean longitudinal momentum balance. Therefore this underestimation may have a minor impact on modelling $\Delta u$ in the lower canopy.

3.4. Scaling arguments and dynamical regions

Because of the multiple length and time scales involved in flow over hills, the mean longitudinal momentum balance can be further simplified using order-of-magnitude analysis and scaling arguments. This simplification can be achieved by decomposing the boundary layer over hills into several distinct regions, each representing a balance between the leading terms in the mean-longitudinal-momentum equation. Such analysis has been successfully used in the last three decades to derive analytical solutions to the mean flow over hills (Jackson and Hunt, 1975; Hunt et al., 1988; Belcher and Hunt, 1993). This approach also forms the basis for numerous simplifications employed in Finnigan and Belcher (2004), which we will now briefly review.

The linear analysis employed by Jackson and Hunt (1975) for flows above hilly terrain leads to two distinct regions: an outer and an inner region. These emerge from time-scale arguments associated with the relative adjustment of the mean and turbulent flow to topographic perturbations. In particular, Belcher and Hunt (1993) introduce the mean-distortion timescale $T_D$ and the Lagrangian-integral time-scale $T_L$ to explore how the mean flow and turbulence adjust to the topographic variations within these two regions. The time-scale $T_D$ characterizes the distortion of turbulent eddies by the straining motion associated with spatial variability in the mean flow induced by the hill. It represents the characteristic time that the mean flow field needs to stretch large eddies through work done by advection against the mean spatial velocity gradients. The time-scale $T_L$ represents the characteristic time for large eddies to come into equilibrium with the local mean flow gradients (Tennekes and Lumley, 1972, chapter 3). The ratio $T_D/T_L$ is used to distinguish the outer and inner regions.

3.4.1. Outer region

When $T_D/T_L \ll 1$, local stretching of large eddies is much slower than distortion due to advection. This region is called the rapid-distortion region, or the outer region, and is characterized by a balance between advection and the pressure-gradient term, with turbulent stresses playing a minor role. Here, the turbulent flow is rapidly distorted, and a direct proportionality between the hill shape and the flow statistics can be assumed using $\Delta x/\Delta u^2 \approx (\Delta u/U_b)(L) = O(H/L)$ (Britter et al., 1981). With these assumptions, the mean-momentum equation becomes:

$$U_b(z) \frac{\partial \Delta u}{\partial x} + \Delta u \frac{\partial U_b(z)}{\partial z} = -\frac{\partial \Delta p}{\partial x}.$$ 

The analytical solution to this equation imposes the upper boundary condition on the inner region.

3.4.2. Inner region

When $T_D/T_L \gg 1$, local stretching of large eddies is fast enough to compete with the distortion due to advection by the mean flow. This region is known as the local-equilibrium region, or the inner region, because local eddies relax to an equilibrium with the local mean velocity gradient before spatial advection can transport and stretch them. This equilibrium permits the use of K-theory to predict perturbations in the turbulent stresses. Spatially, the inner region is defined for $z < h_i$, where $h_i$ is the ‘inner-layer depth’ (Jackson and Hunt, 1975).
In this region, the perturbation stress gradient still plays a minor role in the mean-longitudinal-momentum budget equation, compared with the longitudinal advection, but it is not negligible. Thus, in the inner region the mean flow is governed by both advection and Reynolds stresses. Using first-order-closure models for the inner layer, and the above arguments, we can reduce the linearized mean-momentum equation to:

\[ U_b(z) \frac{\partial \Delta u}{\partial x} + \Delta w \frac{\partial U_b(z)}{\partial z} = -\frac{\partial \Delta p(x)}{\partial x} + 2k_w u \left( \frac{\partial \Delta u}{\partial z} + (z + d) \frac{\partial^2 \Delta u}{\partial z^2} \right). \]

### 3.4.3. Canopy layer

The presence of a canopy sub-layer can alter the lower boundary condition for the inner layer (for example, the no-slip assumption no longer holds, because of the porosity of the canopy), and introduces a drag-force term in the mean-momentum budget equation. Finnigan and Belcher (2004) divide the canopy layer into two distinct sub-layers: an upper canopy sub-layer, in which advective terms are neglected (except through their influence via an asymptotic matching with the inner layer) and \( F_c \) retains only a linear term; and a lower canopy sub-layer, close to the forest floor, in which the balance is between the mean pressure gradient and the nonlinear drag force. The dynamical reason for neglecting the advection terms, in both the upper and the lower canopy sub-layers, is primarily based on the following length-scale argument: vertical variations of the shear stress scale with the mixing length \( l \), and streamwise variation due to drag scales with \( L_c \), while longitudinal variations due to advection scale with the much greater length-scale \( L \). Using these simplifications, Finnigan and Belcher (2004) show that the linearized longitudinal-momentum budget equation can be written as

\[ -\frac{\partial \Delta p}{\partial x} = -2\beta^2 U_b \frac{\partial \Delta u}{\partial z} + 2 \frac{U_b|\Delta u|}{L_c} \]  

in the upper canopy, and

\[ -\frac{\partial \Delta p}{\partial x} = \frac{u |u| - U_b^2}{L_c} \]  

in the lower canopy.

The assumption that advection remains small breaks down if either the length-scale \( L \) becomes short enough that it is comparable with the canopy drag length-scale \( L_c \) (as suggested by Figure 1), or nonlinear processes lead to changes in the mean flow that are occurring on spatial scales smaller than \( L \). Poggi and Katul (2007a) and Ross and Vosper (2005) have demonstrated that for narrow hills, when \( L \approx L_c \), advection is important in the canopy. In particular, Poggi and Katul (2007a) show that advection may be significant not only above the canopy (as predicted by Finnigan and Belcher (2004) and Jackson and Hunt (1975) for the inner layer), but also in the deeper layers of the canopy (especially near the hill summit).

Here, the data are used to explore how much of the spatial variability in measured \( u \) can be explained by a model that retains longitudinal advection in the deeper layers but neglects the turbulent-shear-stress gradient (Regime V). The height of the canopy remains small compared with the length of the hill, so by the usual thin-layer approximation, vertical advection should be small. The presence of a recirculation region may produce substantial vertical advection near the separation and reattachment points, but for an initial treatment this is ignored. A simplified budget equation for such a balance is then given by (3), with the streamwise-advection term reinstated:

\[ \frac{\partial u}{\partial x} = -\frac{\partial \Delta p}{\partial x} - \frac{u |u| - U_b^2}{L_c}. \]  

In essence, this model assumes that inside the canopy (for Regime V), the bulk spatial features of \( u \) are, in a first-order analysis, explained by the mean inviscid equation (Belcher et al., 2007). Turbulence locally adjusts and modulates the mean velocity via an effective turbulent viscosity, but does not play a prominent role in producing and shaping its coarse-grained spatial patterns across the hill. The data of Poggi and Katul (2007a) provide some experimental justification for this argument: their measured turbulent-shear-stress profiles do not vary significantly in the longitudinal direction deep inside the canopy (at least when compared to the layers above the canopy), and show only minor vertical variations in those deeper canopy layers. Loosely speaking, the inviscid dynamics involve a competition between the pressure forces (set by topography here) and the body forces (set by the drag force here) in the absence of any effective viscous (molecular and turbulent) effects and physical boundaries responsible for turbulent production (such as the ground).

### 3.5. Analytical models

Here we present the analytical solutions to the linearized equations within the inner and canopy layers. Appendix B gives details about matching the solutions across these layers, and further simplifications employed in their mathematical derivation.

#### 3.5.1. Inner layer

For flow above a cosine hill, the inner-layer solution for the perturbed longitudinal velocity (which we denote by a tilde) is:

\[ \tilde{u}(x, z) = U_b U_0 \left\{ \left[1 + \log \left(\frac{b^2}{z_0} \right) \right] - 1 \right\} \cos(kx) \]

\[ -\Re \left\{ \frac{\tilde{A}_1 B_0 \exp(ikx)}{\log[(d + z)/z_0]} \right\} \]
where \( U_{io} = (Hk^2/(U^2_i/U^2_0)) \) is a dimensionless scaling for the longitudinal velocity in the inner region, and \( B_0 = 2K_0\sqrt{kL(z + d)/H_i} \), where \( K_0 \) is the modified Bessel function of zeroth order, \( U_i = U_b(h_i) \) is the characteristic velocity in the inner layer, and \( \tilde{A}_1 \) is a constant that allows us to match the velocity in the inner and outer canopy layers (see Appendix B).

Using the continuity equation, we find that the solution for the perturbed vertical velocity above the hill is:

\[
\Delta \tilde{w}(x, z) = \frac{2 \tilde{w}}{kL} \left\{ 2z + \log \frac{h_i^3}{(d + z)z_0} - d \log(d + z) \right\} \sin(kx) - \text{Re} \left( \frac{\tilde{A}_1}{kL} \frac{h_i(d + z)}{B_1} e^{ikx} \right),
\]

where \( B_1 = 2K_1 \sqrt{kL(z + d)/H_i} \), \( K_1 \) being the modified Bessel function of the first kind. The constant \( \tilde{A}_3 \) can be evaluated by imposing the appropriate boundary conditions (see Appendix B).

### 3.5.2. Canopy sub-layer

In the upper canopy, the solution for a cosine hill is given by:

\[
\Delta \tilde{u}(x, z) = -U_b U_c \left\{ \frac{1}{2} \sin(kx) e^{-z/2L_c} + \text{Re} \left( \frac{\tilde{A}_2}{2L_c} e^{ikx} \right) \right\},
\]

where \( U_c = k^2 L_c H(U^2_i/U^2_b) \) is a scaling velocity, and \( \tilde{A}_2 \) can be determined from the boundary conditions (see Appendix B).

Deep within the canopy, the solution is given by:

\[
\Delta u_c(x, z) = \left| U_b^2 - L_c \frac{\partial \Delta p}{\partial x} \right|^{1/2} \text{sgn} \left( U_b^2 - L_c \frac{\partial \Delta p}{\partial x} \right) - U_b.
\]

A single function describing \( \Delta u(x, z) \) within the entire canopy sub-layer can also be derived. Finnigan and Belcher (2004) propose the following phenomenological expression:

\[
\Delta u_c(x, z) = \left| U_b^2 - L_c \frac{\partial \Delta p}{\partial x} \right|^{1/2} \text{sgn} \left( U_b^2 - L_c \frac{\partial \Delta p}{\partial x} \right) + A_2 U_b U_c - U_b.
\]

This expression retains all the primary features of the solutions in the two canopy layers. In particular, the first and second terms on the right-hand side represent the nonlinear response of the flow field to the pressure gradient induced by the hill shape and the turbulent transfer of momentum above the canopy (via \( U_c \)), respectively. Because \( \tilde{A}_2 \) was originally determined to match the perturbed-velocity profile at the canopy top, the uniformly-valid equation for \( \Delta u(x, z) \) no longer matches the perturbed velocity at the bottom of the inner layer.

### 3.5.3. A simple solution

In their numerical analysis of perturbations in CO2 fluxes around a background state defined by 'flat-world micrometeorological conditions', Katul et al. (2006) use a linearized version of \( \Delta u_c \) to generate the mean flow field. This linearized equation is based on the observation that when \( U^2_b \) is much larger or much smaller than \( L_c \theta \Delta p/\partial x \), then \( \Delta u_c \) becomes independent of \( U_b \). Hence, a linearization can be readily carried out, resulting in:

\[
\Delta u_{cl}(x, z) = \left| \frac{L_c}{(\partial \Delta p/\partial x)_{x=L}} \right|^{1/2} \frac{\partial \Delta p}{\partial x} + A_{2L} U_c U_b,
\]

which, for a sinusoidal hill, becomes

\[
\Delta u_{cl}(x, z) = U_c^{1/2} U_b \sin kx + \tilde{A}_{2L} U_c U_b,
\]

where \( \tilde{A}_{2L} \) (see Appendix B) is determined by matching the perturbed longitudinal velocity and the perturbed shear stress at the canopy top with the inner-layer solution.

### 4. Results

In Figure 5 and Table I, the measured \( \Delta u \) and the \( \Delta u \) modelled using the Finnigan–Belcher nonlinear solution (\( \Delta u \)) and the linear solution (\( \Delta u_{cl} \)) are compared by combining all positions within the canopy and inner layers. This ‘global comparison’ reveals that both models exhibit significant correlation with the measurements, but the regression slope is significantly biased away from unity, and the scatter is not small. What is surprising here is that \( \Delta u_{cl} \) predicts the measured \( \Delta u \) better than \( \Delta u \) – at least in terms of the root mean square error. The

#### Table I. Comparison between LDA-measured and modelled velocity perturbations, using both real and virtual grounds to estimate the pressure perturbations.

<table>
<thead>
<tr>
<th>( z/H_c )</th>
<th>( \Delta p )</th>
<th>( r^2 )</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta u_c - \Delta u_m )</td>
<td>&lt; 2</td>
<td>real</td>
<td>0.66</td>
<td>0.64</td>
</tr>
<tr>
<td>( \Delta u_{cl} - \Delta u_m )</td>
<td>&lt; 2</td>
<td>real</td>
<td>0.74</td>
<td>0.52</td>
</tr>
<tr>
<td>( \Delta u_c - \Delta u_{cl} )</td>
<td>&lt; 2</td>
<td>real</td>
<td>0.92</td>
<td>1.26</td>
</tr>
<tr>
<td>( \Delta u_{cont} - \Delta u_m )</td>
<td>&lt; 2</td>
<td>real</td>
<td>0.91</td>
<td>0.89</td>
</tr>
<tr>
<td>( \Delta u_{cl} - \Delta u_m )</td>
<td>&lt; 2</td>
<td>real</td>
<td>0.70</td>
<td>0.63</td>
</tr>
<tr>
<td>( \Delta u_{cl} - \Delta u_{cont} )</td>
<td>&lt; 0</td>
<td>real</td>
<td>0.74</td>
<td>0.43</td>
</tr>
<tr>
<td>( \Delta u_{cl} - \Delta u_{cont} )</td>
<td>&lt; 0</td>
<td>virtual</td>
<td>0.91</td>
<td>0.40</td>
</tr>
<tr>
<td>( \Delta u_{cl} - \Delta u_m )</td>
<td>&lt; 0</td>
<td>virtual</td>
<td>0.91</td>
<td>0.83</td>
</tr>
</tbody>
</table>

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two analytical solutions are also compared to each other in Figure 5; while the scatter in this comparison appears small, the overall difference between them (based on the regression slope) is about 26%.

To explore whether the scatter in the global comparison shown in Figure 5 originates from specific regions or layers, we show each measured and modelled $\Delta u$ profile separately along the hill surface in Figure 6. In addition to $\Delta \tilde{u}_c$ and $\Delta \tilde{u}_{cl}$, $\Delta u$ (upper-canopy solution) and $\Delta u_l$ (lower-canopy solution) are shown for reference. Note that $\Delta \tilde{u}_c$ matches $\Delta u$ in the upper canopy layer and $\Delta u_l$ in the lower canopy layer. We point out three results shown by Figure 6.

- As expected, the nonlinear solution for the entire canopy, $\Delta \tilde{u}_c$, captures the main spatial features of the $\Delta u$ profiles, as well as the transition between them. Nevertheless, the agreement with the measurements remains poor in many regions except on the lee side of the hill.
- The linear solution for the entire canopy, $\Delta \tilde{u}_{cl}$, outperforms $\Delta \tilde{u}_c$ in almost all regions, especially close to the ground and the downwind regions of the hill. The values of $\Delta \tilde{u}_{cl}$ clearly diverge from $\Delta u_l$ in the deeper layers of the canopy, suggesting that the linearized equation drastically differs from the lower-canopy solution proposed by Finnigan and Belcher (2004).

Figure 5. Global comparison between measured $\Delta u_m$ and modelled $\Delta u$ (in ms$^{-1}$) for the nonlinear canopy model ($u_c$) (left) and for the simplified canopy model ($u_{cl}$) used by Katul et al. (2006) (middle). The comparison between $u_{cl}$ and $u_c$ is also shown (right). The one-to-one lines are also presented for reference (regression statistics are presented in Table I). This figure is available in colour online at www.interscience.wiley.com/qj

Figure 6. Profile comparison between measured $\Delta u_m$ (closed circles) and modelled $\Delta u$ (in ms$^{-1}$), for the nonlinear canopy model ($u_c$, dashed lines) and the simplified canopy model ($u_{cl}$, solid lines). The upper-canopy solution $\Delta u_l$ and the lower-canopy solution $\Delta u_l$ are also shown as references (grey solid lines). All the models are driven by the pressure perturbations derived from topography only. The horizontal dashed and dash-dotted lines show the inner-layer and the canopy sub-layer depth, respectively. The locations of the 10 sections are labelled s1–s10, and their locations with respect to the hill are presented in the central panel. This panel shows the spatial patterns of measured $\Delta u$. This figure is available in colour online at www.interscience.wiley.com/qj

• The poorest agreement between Δ\(\tilde{u}\)\(_{cl}\) and the data is in a region around the hill summit. None of the models captures the measured over-speeding in Δ\(\tilde{u}\) near the ground within this region. This suggests that advection can be important, as suggested by Ross and Vosper (2005) and Figure 1.

Figures 7 and 8 repeat the analysis of Figures 5 and 6, but for Δ\(\tilde{w}\); Table I also presents the relevant regression comparisons. Here, the measured Δ\(\tilde{w}\) and its estimated value from the continuity equation Δ\(\tilde{w}_\text{cont}\), and the measured Δ\(\tilde{u}\) assuming no slip at the ground, are also shown. The reason we show both measured and estimated Δ\(\tilde{w}\) is the small magnitude of Δ\(\tilde{w}\), which can be comparable to measurement errors near the ground and within the canopy. From Figures 7 and 8, we conclude the following.

• The LDA-measured Δ\(\tilde{w}\) and the Δ\(\tilde{w}\) estimated from the continuity equation are in good agreement. This lends confidence to both the measured Δ\(\tilde{w}\) and the measured Δ\(\tilde{u}\) inside the canopy.
• The agreement between modelled and measured Δ\(\tilde{w}\) is better than for Δ\(\tilde{u}\), because Δ\(\tilde{w}\) is forced to be zero at the boundary, and because the vertical velocity is largely a response to inviscid forces.

We now focus on two issues pertinent to Regime V and the objectives of this study: the shift in the pressure
gradient; and the role of the two advection terms within the canopy. These two simplifications are explored in Figures 9 and 10, using the analytical solution of Finni-gan and Belcher (2004) but driven by the linear pressure gradient imposed by the ‘effective ground’ and by the simplified inviscid analytical model.

4.1. Pressure-gradient adjustments

Panels (a) and (b) of Figure 9 present the ‘global’ comparison between measured and modelled $\Delta u$ using $\Delta \tilde{u}_{cl}$ for the canopy layer only. Both models are now forced by the pressure perturbations computed from the virtual ground shown in Figure 3. The values of $\Delta \tilde{u}_{cl}$ forced by the pressure perturbations in phase with topography are shown in Figure 9(c) (also for $z/H_c < 0$). Interestingly, the coefficient of determination $r^2$ between measured $\Delta u$ and $\Delta \tilde{u}_{cl}$ significantly improves, suggesting that such a pressure adjustment can have a ‘first-order’ impact on the Finnigan–Belcher solution (see Table I), as anticipated for Regime V. But, as anticipated

![Figure 9. Global comparison, for within-canopy flows ($z/H_c < 0$), between measured $\Delta u_m$ and modelled $\Delta u$ (in ms$^{-1}$). Panels (a) and (c) are for the simplified canopy model $\Delta u_{cl}$, and panel (b) is for the purely advective model $\Delta u_{adv}$. In panels (a) and (b), the models are forced by the pressure perturbations computed from the virtual ground shown in Figure 3, while in panel (c) the modelled $\Delta \tilde{u}_{cl}$ is forced by the theoretical pressure perturbations (in phase with the topography). Data–model comparisons for in-canopy flow are shown for emphasis only. This figure is available in colour online at www.interscience.wiley.com/qj](image1)

![Figure 10. Profile comparison between measured $\Delta u_m$ (closed circles) and modelled $\Delta u$ (in ms$^{-1}$), for the purely advective model $\Delta u_{adv}$ (dash-dotted line) and the linear canopy model $\tilde{u}_{cl}$. For reasons of comparison, $\tilde{u}_{cl}$ is driven by the pressure perturbations computed both from the virtual ground (solid lines) and from the topography (dashed lines). This figure is available in colour online at www.interscience.wiley.com/qj](image2)
from Figure 1, advection is also likely to be a key processes.

4.2. Advection

Recall that an important assumption in Finnigan and Belcher (2004) is that the advective terms are small in relation to the shear-stress gradients, except near the canopy top. Here, they are accounted for through the matching function between the inner layer and the upper canopy layer. The canonical shape of this matching function is a rapid exponential decay within the canopy (Appendix B). Poggi and Katul (2007a) and Ross and Vosper (2005) show that advection cannot be neglected around the hill summit for gentle narrow hills (as in Regimes III and V), and this matching function cannot compensate for the actual advection occurring in this zone.

To address this issue, the measurements are compared with the results from the inviscid model developed in Equation (4), which balances the pressure gradient and drag force with the streamwise advection. In the case of a sinusoidal hill, the solution of this first-order differential equation is found by solving for the kinetic energy of the flow, $E = \frac{1}{2} \bar{u}^2$, which is given by:

$$\tilde{E} = \frac{1}{2}(\Delta \bar{u} + U_b)^2$$

$$= \frac{1}{2} \left( \frac{\alpha}{1 + \alpha^2} \right)^{1/2} \frac{U_b^2}{H} k \sin(kx - \phi) + \frac{U_b^2}{2} + Ce^{-2x/L},$$

where $\alpha = \frac{1}{2} \frac{k}{L_c}$ measures the strength of the advective correction, and gives rise to a downwind phase shift $\phi$ in the solution, where $\tan \phi = \alpha$. The integration constant $C$ is zero because of periodicity in the $x$-direction. In the upper canopy, where the perturbations are small compared to the undisturbed profile, this reduces to:

$$\Delta \bar{u}_{adv} = \frac{U_b U_c}{U_b(z)} \frac{\pi}{16} \frac{\alpha}{(1 + \alpha^2)^{1/2}} \sin(kx - \phi).$$

Note that this solution cannot be matched with the Finnigan–Belcher solution above the canopy, as shear stresses are not included here. Rather, this solution is intended to be a heuristic analysis to diagnose how much of the variability in $\Delta u$ inside the canopy can be explained by the inviscid mean-momentum equation within Regime V.

Figures 9 and 10 compare $\Delta u_{adv}$ with measured $\Delta u$ and $\Delta u_{CL}$ within the canopy (see Table I for regression statistics). Note that the advective solution correctly describes the measured $\Delta u$ in regions where advection is identified as important (Figure 10), such as near the hill top (e.g. Poggi and Katul, 2007a; Ross and Vosper, 2005). Not surprisingly, in regions dominated by the shear-stress term (e.g. $s3$–$s5$ in the upper canopy), the model fails to describe the measured $\Delta u$. The purely adective model also predicts a recirculation region in the deeper layers of the canopy, but with almost the same spatial extent as those predicted by Finnigan and Belcher (2004).

5. Conclusions

Flow over hilly, forested terrain can be divided into different regimes, depending on the ratios $L/L_c$ (the length of the hill compared to the adjustment length-scale) and $H/L_c$ (the depth of the canopy compared to the adjustment length-scale). The dynamical consequences of these regimes are presented in Figure 1. Detailed velocity measurements from a laboratory experiment are aimed at examining Regime V, the most difficult regime because ground shear stress, interactive pressure and advection all affect the velocity perturbations. The experiments are conducted within a short (though dense) canopy ($H/L_c \approx 0.125$) on a gentle ($H/L \approx 0.1$) but narrow ($L/L_c \approx 1$) train of hills for very high bulk Reynolds number ($Re_b > 1.3 \times 10^5$) but low bulk Froude number ($Fr_b < 0.1$). Using these measurements, we have explored the scaling arguments and assumptions of a recently-proposed analytical model developed by Finnigan and Belcher (2004). We come to the following conclusions:

- The often-criticized linearization of the advective terms employed in analytical models such as that of Finnigan and Belcher (2004) appears reasonable except in the very deep layers of the canopy, where the sum of the linearized advective terms underestimates the measured sum by more than 50%. Similarly, the linearized drag force appears to represent well the overall drag except in the lower canopy layers, where $\Delta u$ is large and $U_b$ is small. The linearized shear stress matches well the nonlinear form in the inner layer and the upper canopy layers, however, in lower canopy layers, it is clear that the linearized shear stress significantly underestimates the magnitude of its nonlinear counterpart, by a factor of 4. The data in these regions suggest that the shear stress is small in relation to the other terms in the mean-longitudinal-momentum balance. Hence, this underestimation may have a minor impact on modelling $\Delta u$ in the lower canopy regions.
- The existence of a separation region of reversed flow within the lower canopy on the lee slope agrees with the predictions of the Finnigan–Belcher model. However, the spatial extent of this region appears to be smaller than predicted by that model, and agrees better with predictions made by the simplified version of it.
- The outer-layer pressure perturbations can be adjusted by a virtual ground that accounts for the mean streamline distortions induced by this recirculation zone. The virtual ground permits the pressure in the Finnigan–Belcher model to be readily adjusted. This revision does improve the comparisons between model and
measurements, but the scatter remains significant, hinting that pressure alone is not the only reason why data and model diverge (as suggested by Figure 1).

- The advection in the upper canopy can be important, and must be retained in the mean-momentum balance for the experiments conducted here. Although the experiment lies in Regime V, where we expect the pressure to be interactive, advection is far more important within the canopy. In the deeper layers of the canopy, advection can be the largest term in the mean-momentum balance. An analytical model has been developed that retains a balance between horizontal advection, mean pressure gradient and drag force, but neglects the shear-stress gradient. When model predictions are compared to measured and Finnigan–Belcher-modelled mean velocity perturbations, this model performs no worse inside the canopy across the entire hill, and performs better than the Finnigan–Belcher model in the deeper layers of the canopy, confirming that advection can be important for Regime V (see Figure 1). Hence, the main conclusion from this analysis is that the inviscid solution to the mean-momentum budget appears to explain much of the spatial structure in $\Delta u$ with turbulence (via the turbulent-stress gradient), locally modulating some of it through the action of a turbulent viscosity in Regime V (see Figure 1). The Finnigan–Belcher model does indirectly account for advection, through a matching function between the upper-canopy solution and the inner layer. However, this matching function is known to decay exponentially within the canopy, and plays a minor role (except near the canopy top).

- The agreement between modelled and measured $\Delta w$ is better than for $\Delta u$. This may be because the lower boundary condition ($\Delta w = 0$) is shared between data and all the models.

From a broader perspective, canopy lidars, such as SLICER (Lefsky et al., 2002), now provide detailed measurements of topographic variability, canopy height, and leaf-area density. What is clearly missing is a simplified framework that links mean-velocity perturbations to topographic variations derived from such remote-sensing products. This study has shown that when we are dealing with complex yet gentle terrain, the validity of the linearized equations implies that the total perturbations in the mean pressure can be determined from superposition of the resulting perturbations induced by the individual modes in the topography. A consequence of this finding is that even if the available topographic information does not follow a continuous function with desired spectral properties, a Fourier decomposition can still be employed on the discrete lidar data, along with superposition of pressure perturbations.

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**A. Appendix: Flow regimes**

As stated in Section 1, the Finnigan–Belcher model is applicable to deep canopies on gentle and long hills, assuming that the pressure in the outer layer is not affected by the flow inside the canopy. Figure 1 shows a number of plausible flow regimes inside canopies on gentle hilly terrain ($H/L = 0.1$), depending on the following characteristic length-scales: the hill height $H$, the hill half-length $L$, the canopy adjustment length-scale $L_c$, and the canopy height $H_c$. From these variables we can derive three dimensionless parameters $H/L$, $H_c/L_c$ and $L/L_c$—each of which can be used to classify case studies into flow regimes, as follows.

**Deep and shallow canopies** The first classification is based on the density of the canopy. Using an exponential mean-velocity profile inside the canopy, one can derive a threshold for deep canopies using one of two arguments.

- The shear length-scale has been widely used for describing turbulence in the canopy layer (Raupach et al., 1996). It has been empirically shown that for a dense canopy, $H_c/L > 1$ (Brunet and Irvine, 2000; Finnigan, 2000; Poggi et al., 2004a): $L_s = U_h/(dU_h/dz)_{z=0}$ can be expressed as $L_s = l_{c1}/\beta$ or $L_s = 2\beta^2 L_c$. From this simple derivation, it is reasonable to define ‘deep’ canopies as those where $H_c/L_c = 2\beta^2$.

- In a dense canopy, the entire momentum is absorbed by the foliage, so that the stress at the ground is negligible. Thus the ratio of the shear stress at the base of the canopy to the shear stress at the canopy top can be taken as $\exp(-H_c/\beta^2 L_c) < 0.1$, which immediately yields $H_c/L_c > 2\beta^2$.

Note that, following the terminology of Finnigan and Belcher (2004) and Ross and Vosper (2005), but not that of Poggi et al. (2004a, b, c), we refer to ‘deep’ and ‘shallow’ canopies instead of ‘dense’ and ‘sparse’ canopies.

**Narrow and long hills** The second classification is based on the ratio between the hill half-length and the canopy adjustment length-scale. Long hills are defined as having a ratio $L/L_c > 2$; narrow hills are those where $L/L_c < 2$.

**Interactive and fixed pressure** The boundary shown in Figure 1 delineates conditions where the pressure...
is primarily determined by the hill shape (the fixed-pressure regime) and where the flow within the canopy significantly changes this pressure (interactive-pressure regime). As discussed in Finnigan and Belcher (2004), the pressure field develops in the outer region above the canopy through inviscid irrotational dynamics, so that the pressure \( p \) is related to the near-surface vertical velocity \( w_0 \) by

\[
p = p_0 e^{-kz},
\]

where

\[
p_0 = iw_0U_0.
\]

The vertical velocity \( w_h \) forced by the topography is

\[
w_h = iakU_0e^{ix}.
\]

The flow within the canopy also leads to a vertical velocity at the top of the canopy, denoted here by \( w_c \). The model for the flow above the canopy is linear, and so the total vertical velocity that forces the pressure is the sum \( w_0 = w_h + w_c \). The pressure is primarily fixed by the topography when \( w_0 \gg w_c \). The \( w_c \) term can be estimated as follows. Finnigan and Belcher (2004) show that at the crest of the hill there is convergence of the flow within the canopy, which leads to a vertical motion there. Consider a small box, of height and width \( h \), drawn around the canopy at the crest of the hill. The magnitude of the vertical velocity \( w_c \) at the crest can then be estimated by balancing the horizontal flux into the box with the vertical flux out of the top, which is of order \( hw_c \). The horizontal wind speed deep in the canopy is

\[
U = \text{sgn}(-L_c p_0')L_c p_0^{1/2}.
\]

At the crest, \( p_0' = 0 \); and at a small distance \( h \) from the crest, \( p_0' \approx h p_0'' \), so that the vertical velocity is given by the horizontal flux into the box, according to:

\[
w_c \approx \frac{1}{h} \int_0^h u \, dz \approx h^{1/2}L_c p_0^{1/2}.
\]

The boundary between the fixed- and interactive-pressure regimes occurs when \( w_c \), given by Equation (8), is comparable to \( w_c \), given by Equation (9). Since \( p_0'' \approx U_0^2 H/L^3 \), this provides the condition used in Figure 1, namely:

\[
\frac{H_c}{L_c} \approx \frac{H}{L} \left( \frac{L}{L_c} \right)^2.
\]

These three criteria are used to define the five regimes in Figure 1, although several other regimes can be identified. The five regimes are as follows.

**Regime I** The pressure behaviour is imposed by the topography. Hills are long, so that advection remains small; and the canopy is deep, so that the momentum is entirely absorbed by the foliage elements.

**Regime II** Hills are still long, and the canopy is still deep (or dense). However, the vertical velocity is large enough to affect the pressure behaviour.

**Regime III** Neither advection nor pressure interactions can be neglected. However, the canopy density is still large enough (or the canopy is sufficiently deep) to absorb the entire momentum.

**Regime IV** The hills are long, and the pressure is primarily determined by the topography, but the canopy is now sparse (or shallow).

**Regime V** None of the above assumptions about canopy density, long hills or fixed pressure hold.

Figure 1 shows that the data are in Regime V. While advection, pressure interaction and ground shear stress cannot be entirely neglected here, this study explores whether one of them primarily explains the departures from the Finnigan–Belcher model (or Regime I).

### B. Appendix: Uniform solution for the canopy: linear and nonlinear models

To obtain a uniform solution for the entire canopy, Finnigan and Belcher (2004) propose a phenomenological model,

\[
\Delta u_c(x, z) = \left| U_b^2 - L_c \frac{\partial \Delta p}{\partial x} \right|^{1/2} \text{sgn} \left( U_b^2 - L_c \frac{\partial \Delta p}{\partial x} \right) + A_2 U_c e^{1/2} \cos k x - U_b,
\]

which retains the primary features of the solutions in the two canopy layers, yet provides a realistic transition between them. Katul et al. (2006) use a linearized version of the above equation, resulting in

\[
\Delta u_{cl}(x, z) = \frac{L_c}{\left( \partial \Delta p/\partial x \right)_{x=L}} \left| \frac{\partial \Delta p}{\partial x} \right|^{1/2} + A_{2L} U_c U_b,
\]

where, for a sinusoidal hill, becomes:

\[
\Delta \tilde{u}_{cl}(x, z) = U_c^{1/2} U_b \sin(kx) + \tilde{A}_{2L} U_c U_b,
\]

where \( \tilde{A}_{2L} \) must be determined by matching the perturbed longitudinal velocity and the perturbed shear stress at the canopy top with the inner-layer solutions.

The continuity equation and \( \Delta \tilde{u}_{cl} \) can be used to derive \( \Delta \tilde{w}_{cl} \), given by

\[
\Delta \tilde{w}(x, z)_{cl} = U_b U_c^{1/2} k (H_c + z) \cos(kx),
\]

where the boundary condition \( \Delta \tilde{w}_{cl}(x, 0) = 0 \) is imposed.

From \( \Delta \tilde{u}_{cl} \), the stress perturbation is given by:

\[
\Delta \tilde{T}(x, z) = 2\beta^2 U_b^2 U_c^{1/2} \tilde{A}_{2L}.
\]
Note that in Equation (10), only the turbulent transfer of momentum from above the canopy contributes to $\Delta \tau (x, z)$. This means that the primary difference between the linear and nonlinear solutions for $\Delta \tau (x, z)$ is that, while both take into account the turbulent transfer of momentum from above the canopy, only the nonlinear solution is directly sensitive to the pressure gradient induced by the hill shape. The determination of the constants is discussed next.

B.1. Constants for the nonlinear solution

The coefficients $\tilde{A}_1$ and $\tilde{A}_2$ are evaluated by matching the solutions for $\Delta \tilde{u}$ and $\Delta \tau$ at the top of the canopy. This procedure yields:

$$\tilde{A}_1 = \frac{1 - \log \frac{d}{h_i} + \delta (1 - \frac{i k L c U_i}{H c}) + \frac{k_u u_*}{\beta^2 U_h}}{B_0 + \frac{2 B_1 d k L c u_*}{\beta^2 h_i U_h}}$$

and

$$\tilde{A}_2 = \left[ \frac{Ar k_v}{2} \left( 1 - \frac{d}{h_i} + \delta \left( 1 - \frac{i k L c U_i}{2 U_h} \right) \right) \right]$$

$$+ \frac{B_0 \left( \frac{\beta^2 L c U_i}{2 u_*} - \frac{k_u}{h_i} \right)}{B_1} \cdot \left[ \delta L_c \left( \frac{U_i}{u_*} \left( \frac{B_0}{B_1} \beta^2 + \frac{1}{2} Ar k_v u_u U_h \right) \right)^{-1} \right].$$

where $\delta = \log (h_i/z_0)$, and $Ar = 2 \sqrt{i k L (z + d)}/h_i$ is the argument of both the modified Bessel functions of zeroth and first order ($B_0$ and $B_1$).

The coefficient $A_3$ in Equation (5) has to be evaluated for $\Delta \tilde{u}$. This is accomplished by imposing $\Delta \tilde{u} = 0$ at the ground and matching $\Delta \tilde{u}$ and $\Delta \tilde{u}_a$ at the top of the canopy. This procedure yields:

$$\tilde{A}_3 = H k^3 L c^2 \frac{U_i^2}{U_h} \left[ \frac{\beta^2}{2} \left( e^{i \theta H L c / i} + 2 i \tilde{A}_2 (e^{i \theta H L c / i} - 1) \right) - \frac{1}{2 i A_2 \tilde{A}_1 \frac{U_i}{u_*}} \right] \frac{e^{i k x}}{2 k L c^2 U_i}.$$

B.2. Constants for the linear solution

The unknown coefficient in the linear solution for $\Delta \tilde{u}_a$ is $\tilde{A}_{2L}$. The coefficient in the inner-layer solution for $\Delta \tilde{u}$, hereafter denoted $\tilde{A}_{2L}$, must also be modified to match $\Delta \tilde{u}_a$ at the canopy top. Upon matching the solutions for the velocity and stress above the canopy and inside the canopy and Equation (10) at the canopy top, we obtain

$$\tilde{A}_{1L} = \frac{1 - \log \frac{d}{h_i} + \delta (1 - i \sqrt{\frac{2 L c U_i}{H U_0}}) + \frac{k_u u_*}{\beta^2 U_h}}{B_0 + \frac{2 B_1 d k L c u_*}{\beta^2 h_i U_h}}$$

and

$$\tilde{A}_{2L} = -\frac{2 B_0}{B_1} + \frac{1 - \log \frac{d}{h_i} + \delta \left( 1 - i \sqrt{\frac{2 L c U_i}{H U_0}} \right)}{k \delta L_c (Ar U_i + U_i \frac{2 B_1 \beta^2}{U_h} \frac{B_i k_v}{U_h})}.$$

As for the nonlinear solution of $\Delta \tilde{u}$, we must determine the coefficient in Equation (5), now called $\tilde{A}_{3L}$, to obtain a continuous solution for $\Delta \tilde{u}$. As before, this is achieved by imposing $\Delta \tilde{u}_a = 0$ at the ground in Equation (10), and matching $\Delta \tilde{u}$ and $\Delta \tilde{u}_a$ at the top of the canopy. This yields:

$$\tilde{A}_{3L} = \frac{U_i^2}{2 U_h} \left( \frac{h_i}{L} \delta \left( \frac{\tilde{A}_{1L} \frac{Ar U_i}{2} + \delta k L c U_i}{2 U_h} \sqrt{\frac{L c}{2 H U_0}} \right) \right) - \frac{i e^{i \theta H L c / i} k \delta L c}{\beta} \left( - \frac{U_i}{U_h} + \frac{U_i}{U_h} e^{i \theta H L c / i} \right) \frac{e^{i k x}}{2 k L c^2 U_i}.$$

In this appendix we have demonstrated how to relate all the integration constants to the hill-shape, canopy and mean-flow attributes.

References


