Dissipation methods, Taylor’s hypothesis, and stability correction functions in the atmospheric surface layer

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Abstract. The traditional dissipation method and the new approaches suggested by Albertson et al. [1996] and Hsieh et al. [1996] to estimate momentum and heat fluxes were compared using velocity and temperature measurements in the atmospheric surface layer. These measurements were carried out above two different sites (grass and bare soil) over a wide range of atmospheric stability and turbulent intensity conditions. Taylor’s hypothesis, flux divergence terms, and stability correction functions which play important roles in the dissipation methods were also evaluated. In highly turbulent intensity flows, deviations from Taylor’s hypothesis may cause some errors in estimating dissipation rates and subsequent fluxes. Wyngaard and Clifford [1977] proposed a model to interpret the influence of departures from Taylor’s hypothesis. In this study we evaluate this influence in the inertial subrange and discuss the usefulness of Wyngaard and Clifford’s model in practice. We also found the flux divergence term in the temperature variance budget equation to be significant, relative to the production term in unstable conditions. Our measurements showed that discarding the flux divergence term resulted in systematic underpredictions of the sensible heat flux by the dissipation methods. The proposed dissipation method by Hsieh et al. [1996] for estimating sensible heat flux was extended to momentum flux, and its implications for stability correction functions were discussed. Good agreement between eddy correlation measured and predicted sensible heat fluxes by the methods of Albertson et al. [1997] and Hsieh et al. [1996] was noted.

1. Introduction

Estimating land surface fluxes using the dissipation method continues to be an active research topic [e.g., Edson et al., 1991; Marsden et al., 1993; Albertson et al., 1996] in boundary layer meteorology, ocean-atmosphere interactions, and surface hydrology since this method is less sensitive to instrument platform motion, sensor alignment and orientation, and stringent steadiness in the mean meteorological conditions when compared to eddy correlation methods. However, the dissipation method suffers from the following limitations: (1) the validity of Taylor’s [1938] hypothesis is assumed for estimating the mean dissipation rates of turbulent kinetic energy and scalar variances, and (2) the atmospheric stability correction functions for momentum (\(\phi_m\)), heat (\(\phi_h\)), and other scalars or the normalized functions of dissipation rates are utilized despite uncertainty in the derivation or measurements of such functions.

The first limitation is attributed to the application of Taylor’s [1938] hypothesis to convert time domain to space domain. Early experiments by Fisher and Davies [1964] clearly showed that small-scale eddy convection velocity is variable and differs from the mean longitudinal velocity when the turbulent intensity is large. Champagne et al. [1977] also concluded that deviations from Taylor’s hypothesis for highly turbulent intensity flows have significant effects on determining the dissipation rates from energy spectra. A model for interpreting high-frequency longitudinal velocity spectra in highly turbulent intensity flows was derived by Lumley [1965] and later extended to lateral velocity and scalar spectra by Wyngaard and Clifford [1977] for isotropic and anisotropic homogeneous Gaussian velocity fluctuations. While the theoretical developments of Wyngaard and Clifford [1977] are constrained by the Gaussian velocity approximation, such models have been employed in field experiments with mixed conclusions regarding improved dissipation estimates [e.g., Bradley et al., 1981].

The second limitation is attributed to the availability of several atmospheric stability correction functions (for reviews, see Högström [1996], Kader and Yaglom [1980], and Hsieh et al. [1996]) with varying degrees of accuracy, asymptotic limits, and stability ranges. The uncertainty in such functions introduces an added uncertainty in the fluxes estimated by the dissipation method. However, the dissipation methods proposed by Albertson et al. [1996] and Hsieh et al. [1996] for estimating sensible heat flux avoid this uncertainty. It is these limitations and the preliminary success of the methods proposed by Albertson et al. [1996] and Hsieh et al. [1996] that have motivated this study.

The objectives of this study are as follows. (1) to compare flux estimation by the traditional dissipation method and the methods proposed by Albertson et al. [1996] and Hsieh et al. [1996] with direct eddy correlation measurements, (2) to examine the influence of stability correction functions on the dissipation methods, and (3) to evaluate the influence of deviation from Taylor’s hypothesis in the relevance of flux calculations by dissipation methods and to examine Wyngaard and Clifford’s [1977] model.

2. Theory

In this section, theories of the traditional dissipation method and the methods proposed by Albertson et al. [1996] and Hsieh et al. [1996] are reviewed. Then Wyngaard and Clifford’s [1977] model is discussed.
2.1. Traditional Dissipation Method and Budget Equations

The traditional dissipation method is based on the turbulent kinetic energy (TKE) and the scalar variance \( \left( e.g. \text{, temperature, water vapor, CO}_2 \right) \) budget equations in conjunction with the Monin and Obukhov [1954] similarity theory (hereafter referred to as MOST). In a stationary and horizontally homogeneous atmospheric surface layer (ASL), the budget equations for TKE and temperature variance are

\[
-\langle \omega w \rangle \frac{\partial U}{\partial z} + \frac{g}{T} \langle w \theta \rangle = -\frac{1}{2} \frac{\partial (\omega e^2)}{\partial z} + \frac{1}{\rho} \frac{\partial (wp)}{\partial z} = e \tag{1a}
\]

\[
-\langle w \theta \rangle \frac{\partial T}{\partial z} = \frac{1}{2} \frac{\partial (w \theta^2)}{\partial z} = N_o \tag{1b}
\]

where \( U, v, w, \) and \( \theta \) are the fluctuations of longitudinal, lateral, and vertical wind velocity and air potential temperature, respectively; angle brackets denote averaging, \( z \) is the height above the zero-plane displacement, \( U \) and \( T \) are the mean longitudinal wind velocity and air potential temperature, respectively, \( g = 9.8 \text{ m s}^{-2} \) is the gravitational acceleration, \( e^2 = \langle w^2 \rangle = \langle v^2 \rangle + \langle w^2 \rangle \) is twice the turbulent kinetic energy, \( \rho \) is the mean air density, \( p \) is the static pressure fluctuation, and \( e \) and \( N_o \) are the mean dissipation rates for TKE and \( \langle w \theta \rangle / 2 \), respectively (see Notation section for definitions). In practice, the flux divergence terms in (1a) (see, e.g., Fairall and Larsen [1986] and Panofsky and Dutton [1984] for reviews) and (1b) [e.g., Panofsky and Dutton, 1984; Champagne et al., 1977] are neglected, so that (1a) and (1b) simplify to

\[
-\langle \omega w \rangle \frac{\partial U}{\partial z} + \frac{g}{T} \langle w \theta \rangle = e \tag{2a}
\]

\[
-\langle w \theta \rangle \frac{\partial T}{\partial z} = N_o \tag{2b}
\]

With the definition of \( \phi_m = (kz/\langle u \theta \rangle) \partial (\bar{U} / \partial z) \); \( k = 0.4 \) is the von Karman constant; \( \langle \bar{U} \rangle = \langle \omega w \rangle \langle z \rangle \rangle \) is the friction velocity, \( \phi_o = (\omega \theta / \langle u \theta \rangle) \) (\( \theta \) is a temperature scale) and the Monin-Obukhov stability length, \( L = -\langle u \theta^2 / T \rangle / \langle w \theta \rangle \), (2a) and (2b) lead to the equations

\[
\phi_e = \frac{e}{u_*^3} \tag{3a}
\]

\[
H = \rho C_p \left( \frac{N_o k z u_*}{\phi_o} \right)^{1/2} \tag{3b}
\]

for estimating \( u_* \) and \( H \) in the ASL, where \( C_p \) (1005 J kg\(^{-1}\) K\(^{-1}\)) is the specific heat capacity of dry air at constant pressure.

Also, without neglecting the flux divergence terms in (1a) and (1b), \( u_* \) and \( H \) can be determined from the normalized functions of \( e \) and \( N_o \) which are defined as

\[
\phi_e = \frac{e}{u_*^3} \tag{4a}
\]

\[
\phi_{N_o} = \frac{N_o k z}{u_* \theta_*^{1/2}} \tag{4b}
\]

respectively. Hence, by rearranging (4a) and (4b), \( u_* \) and \( H \) can be estimated from

\[
u_* = \left( \frac{e k z}{\phi_e} \right)^{1/3} \tag{5a}
\]

\[
H = \rho C_p \left( \frac{N_o k z u_*}{\phi_o} \right)^{1/2} \tag{5b}
\]

The relationship between \( \phi_e \) and \( \phi_{N_o} \) and \( \phi_o \) can be obtained by normalizing (1a) with \( k z / u_*^2 \) and (1b) with \( k z / (u_* \theta^{1/2}) \). Namely, compared to (5a) and (5b), (3a) and (3b) assume that \( \phi_m = -z/\theta \approx \phi_e \) and \( \phi_o \approx \phi_{N_o} \), respectively.

If \( e \) and \( N_o \) are known (here \( e \) and \( N_o \) are determined from second-order structure functions as discussed in section 2.4), then (3a) and (3b) or (5a) and (5b) can be solved iteratively [Brutsaert, 1982, p. 194] to determine \( u_* \) and \( H \) with specified empirical functions for \( \phi_o \) and \( \phi_e \), or \( \phi_{N_o} \). Here, the following empirical functions, expressed in terms of stability parameter \( z/L \), are considered in unstable and stable atmospheric surface layers.

In unstable atmospheric conditions, for \( \phi_e \) and \( \phi_{N_o} \), we consider the widely used Businger-Dyer functions [Dyer, 1974], which are

\[
\phi_e = (1 - 16z/L)^{-1/4} \quad -2 < z/L < 0 \tag{6a}
\]

\[
\phi_o = (1 - 16z/L)^{-1/2} \quad -2 < z/L < 0 \tag{6b}
\]

and the empirical functions of Kader and Perpeletkin [1989], since their functions are continuous over a wide range of stability conditions and agree well with the three-sublayer functions proposed by Kader and Yaglom [1990]. With the inclusion of the von Karman constant \( k \), which was omitted from the original expression, these functions become

\[
\phi_e = \frac{1 + 0.625 (-z/L)^7}{1 + 7.5 (-z/L)^2} \tag{7a}
\]

\[
\phi_o = 0.64 \left[ \frac{3 + 2.5 (-z/L)}{1 + 10 (-z/L) + 50 (-z/L)^2} \right]^{1/3} \tag{7b}
\]

For \( \phi_e \) and \( \phi_{N_o} \), we consider Kader's [1992] empirical functions

\[
\phi_e = 0.4 \left[ \frac{10 + 7.5 (-z/L) + 6.25 (-z/L)^2}{4 + 2.5 (-z/L)} \right] \tag{8a}
\]

\[
\phi_{N_o} = 0.4 \left[ \frac{10 + 7.5 (-z/L) + 6.25 (-z/L)^2}{1 + 125 (-z/L)^2 + 78.125 (-z/L)^3} \right]^{1/3} \tag{8b}
\]

The widely used function \( \phi_e = \left[ 1 - 0.5 (-z/L)^{3/2} \right]^{3/7} \) [Wyngaard and Cote, 1971] is not considered since this function is only suitable for a limited stability range \( 0 < -z/L < 2 \) and deviates from (8a) appreciably for \( -z/L > 2 \). It is important to check the comparisions between \( \phi_m = -z/L \) and \( \phi_e \) and \( \phi_o \) and \( \phi_{N_o} \) with the empirical functions described above. Figures 1a and 1b show the comparison and suggest that from these published forms, neglecting the flux divergence terms may be valid in the TKE budget equation (1a), but not valid in the temperature variance budget equation (1b) for moderately and strongly unstable conditions.

In stable atmospheric conditions, less is known about these functions. According to Dyer [1974],
\[ \phi_w = 1 + 5z/L \quad z/L > 0 \]  
(9a)  
\[ \phi_b = 1 + 5z/L \quad z/L > 0. \]  
(9b)

For \( \phi_e \) and \( \phi_{Ne} \), according to Wyngaard and Cote [1971], we assume

\[ \phi_e = \phi_m - z/L = 1 + 4z/L \quad z/L > 0 \]  
(10a)  
\[ \phi_{Ne} = \phi_b = 1 + 5z/L. \]  
(10b)

This assumption leads to (3a) being identical to (5a) and (3b) being identical to (5b).

2.2. Albertson et al.'s [1996] Dissipation Method

In the convective ASL, \( N_u \) should be independent of \( u_\ast \); hence \( \phi_{Ne} \) should scale with \( (z/L)^{-1/3} \) and can be expressed as

\[ \phi_{Ne} = A \left( -z/L \right)^{-1/3} \]  
(11)

where \( A \) is a similarity constant. On the basis of (11) and the definition of \( \phi_{Ne} \), Albertson et al. [1996] derived a relation, given by

\[ H = A^{-3/2} \rho C_p \left( \frac{\theta}{T} \right)^{1/2} (kz)^{1/2} N_u^{5/2} \]  
(12)

for estimating \( H \) in the unstable ASL. Notice that (12) provides a simple method to compute \( H \) without \( \phi_{Ne} \) or \( \phi_b \) and iteration toward the solution. By Kader and Yaglom [1990], (11) can also be derived for dynamic convective conditions, and many field measurements [e.g., Kiely et al., 1996; Albertson et al., 1996, Kader and Yaglom, 1990] also confirm that (11) is suitable for both dynamic convective and convective conditions. Hence it seems suitable to apply (12) to the whole unstable conditions. However, no equivalent formulation can be derived for estimating \( \Pi \) for the stable ASL.

2.3. Hsieh et al.'s [1996] Dissipation Method

On the basis of dimensional analysis and the temperature variance budget equation (neglecting the flux divergence term), Hsieh et al. [1996] proposed that the sensible heat flux is governed by \( N_o \), \( \sigma_{\theta} \), and \( z \) which can be expressed as (see Appendix for details)

\[ H = \rho C_p \frac{k_o N_o z}{\sigma_{\theta}} \]  
(13)

where \( k_o \) is a similarity constant. Notice in (13) that the stability correction function for heat is no longer needed to determine \( H \). Now on the basis of (2a) and the same dimensional analysis in (13), we are able to express the momentum flux as (see appendix for details)

\[ u_\ast^2 = \frac{k_o \left( \frac{e - \frac{g}{T} \left< \theta \theta \right>}{\sigma_{\theta}} \right) z}{\sigma_u} \]  
(14)

where \( k_o \) is a similarity constant and \( \left< \theta \theta \right> \) can be estimated from (13). Hence (13) and (14) can be used to estimate the fluxes of sensible heat and momentum without requiring \( \phi_m \) and \( \phi_b \) (or \( \phi_e \) and \( \phi_{Ne} \)). Using the argument of Hsieh et al. [1996], (13) and (14) lead to (see appendix for details)

\[ k_o \frac{\phi_b}{\phi_b} = k_o \]  
(15a)  
\[ k_o \frac{\phi_m}{\phi_m} = k_o. \]  
(15b)

where \( \phi_b \), \( \phi_m \), and \( \phi_{Ne} \) are the normalized function for \( \sigma_{\theta} \) and \( \sigma_u \) (= \sigma_{\theta}/u_\ast ; \sigma_u = (\left< u^2 \right>^{1/2}) \) is the normalized function for \( \sigma_u \). In other words, (15a) and (15b) indicate that \( \phi_b \) divided by \( \phi_b \) and \( \phi_m \) divided by \( \phi_m \) should be constant, and it is possible to determine \( k_o \) and \( k_o \) from MOST if MOST is valid. By Kader and Yaglom's [1990] three-sublayer model for \( \phi_b \) and \( \phi_m \) in the unstable ASL, Hsieh et al. [1996] calculated \( k_o \) to be between 1.21 and 1.67 (Hsieh et al.'s [1996] data showed \( k_o = 1.66 \)). To estimate \( k_o \), we consider the values of \( \phi_m \) and

![Figure 1a](image-url)  
Figure 1a. Normalized functions of the dissipation rate of turbulent kinetic energy, \( \phi_e \), as a function of \(-z/L\). The solid line denotes \( \phi_e \approx \phi_m - z/L \), where \( \phi_m \) is given as (6a) [Dyer, 1974]. The dashed line denotes \( \phi_e \approx \phi_m - z/L \), where \( \phi_m \) is given as (7a) [Kader and Perpekelin, 1989]. The dotted line denotes \( \phi_e \) as given by (8a) [Kader, 1992].

![Figure 1b](image-url)  
Figure 1b. Normalized functions of the dissipation rate of \( \left< \theta^2 \right>/2 \), \( \phi_{Ne} \), as a function of \(-z/L\). The solid line denotes \( \phi_{Ne} \approx \phi_b \), where \( \phi_b \) is given as (6b) [Dyer, 1974]. The dashed line denotes \( \phi_{Ne} \approx \phi_b \) [Kader and Perpekelin, 1989], where \( \phi_b \) is given as (7b) [Kader and Perpekelin, 1989]. The dotted line denotes \( \phi_{Ne} \) as given by (8b) [Kader, 1992].
\( \phi_n \) for neutral conditions since they are better defined by MOST under these stability conditions. In neutral conditions, \( \phi_m = 1 \) and \( \phi_n \approx 2.7 \) (Kader and Yaglom [1990] note \( \phi_n \approx 2.5 \) for laboratory experiments), thus \( k_n \approx 1.1 \).

Now since the sensible heat flux is negative under stable conditions, more information (e.g., temperature measurements at another height) is necessary to determine the sign of \( H \) estimated from all the dissipation methods.

2.4. Wyngaard and Clifford’s [1977] Model and Determination of \( \varepsilon \) and \( N_a \)

From Kolmogorov’s [1941] theory, \( \varepsilon \) and \( N_a \) are generally determined from the measured structure functions. In the inertial subrange the second- and third-order structure functions for \( u \) and the second-order structure functions for \( v \), \( w \), and \( \theta \) are given by

\[
D_{uu}(r) = \langle [u(x + r) - u(x)]^2 \rangle = 4\alpha \varepsilon^{2/3} r^{5/3} \quad (16a)
\]

\[
D_{vw}(r) = \langle [u(x + r) - u(x)]^2 \rangle = -\frac{4}{3} \varepsilon r \quad (16b)
\]

\[
D_{\theta\theta}(r) = \langle [\theta(x + r) - \theta(x)]^2 \rangle = (4/3)4\alpha \varepsilon^{2/3} r^{5/3} \quad (16c)
\]

\[
D_{ww}(r) = \langle [w(x + r) - w(x)]^2 \rangle = -\frac{4}{3} \alpha \varepsilon^{2/3} r^{5/3} \quad (16d)
\]

\[
D_{\theta\theta}(r) = \langle [\theta(x + r) - \theta(x)]^2 \rangle = 4\beta N_a e^{-10\varepsilon^{2/3}} \quad (16e)
\]

where \( \alpha \) and \( \beta \) are Kolmogorov constants for one-dimensional \( u \) and \( \theta \) spectra and \( r \) is the separation distance of two measurements along the \( x \) axis. For calculating the structure functions, Taylor’s hypothesis (\( r = Ut \)) should be valid in order to convert the time difference, \( t \), into spatial difference, \( r \). However, Taylor’s hypothesis is violated if the turbulent intensity \( I_u = \sigma_u / U \) is large. On the basis of Launder’s [1965] model, Wyngaard and Clifford [1977] developed a model for interpreting high-frequency velocity and scalar spectra in highly turbulent intensity flows. When \( \varepsilon \) and \( N_a \) are determined directly from the time derivatives of \( u \) and \( \theta \), Wyngaard and Clifford’s [1977] model suggests that

\[
\varepsilon = 15\nu \left( \frac{\partial u}{\partial x} \right)^2 = 15\nu \left( \frac{\partial u}{\partial t} \right)^2 F_{\alpha}(I_u) \quad (17a)
\]

\[
N_a = 3\chi \left( \frac{\partial \theta}{\partial x} \right)^2 = 3\chi \left( \frac{\partial \theta}{\partial t} \right)^2 F_{\alpha}(I_u) \quad (17b)
\]

where \( \nu \) is kinematic viscosity, \( \chi \) is the thermal molecular diffusivity, and \( F_{\alpha}(I_u) \), defined as \( 1 + 5I_u^2 \) and \( F_{\alpha}(I_u) \), defined as \( 1 + 3I_u^2 \), are Wyngaard and Clifford’s correction functions due to the application of Taylor’s hypothesis for isotropic and highly turbulent intensity flows. Equations (17a) and (17b) have been used by some investigators [e.g., Champagne et al., 1977; Bradley et al., 1981; Onset et al., 1996]; however, the added accuracy of this correction is still uncertain [Bradley et al., 1981]. If \( \varepsilon \) and \( N_a \) are determined from the structure functions, Wyngaard and Clifford’s model suggests that

\[
D_{uw}(r) = \langle [u(x + r) - u(x)] [w(x + r) - w(x)] \rangle = 4[\alpha F_{\alpha}(I_u)] e^{-2/3} r^{2/3} \quad (18a)
\]

\[
D_{vw}(r) = \langle [v(x + r) - v(x)] [w(x + r) - w(x)] \rangle = (4/3)4[\alpha F_{\alpha}(I_u)] e^{-2/3} r^{2/3} \quad (18b)
\]

\[
D_{\theta\theta}(r) = \langle [\theta(x + r) - \theta(x)] [\theta(x + r) - \theta(x)] \rangle = (4/3)4[\alpha F_{\alpha}(I_u)] e^{-2/3} r^{2/3} \quad (18c)
\]

\[
D_{\theta\theta}(r) = \langle (\theta(x + r) - \theta(x))^2 \rangle = 4[\beta F_{\theta}(I_u)] N_a e^{-10\varepsilon^{2/3}} \quad (18d)
\]

where \( F_{\alpha}(I_u) = 1 + (11/9)I_u^2 \), \( F_{\alpha}(I_u) = F_{\alpha}(I_u) - 1 + (11/36)I_u^2 \), and \( F_{\theta}(I_u) = 1 + (5/9)I_u^2 \) are correction functions for \( u \), \( v \), \( w \), and \( \theta \) structure functions, respectively (these corrections were originally proposed for one-dimensional spectra). In this study we concentrate on the structure functions for calculating \( \varepsilon \) and \( N_a \). In applying Wyngaard and Clifford’s [1977] model, we note the following:

1. In (18a)–(18d), what Wyngaard and Clifford proposed is a correction to the Kolmogorov constants \( \alpha \) and \( \beta \), rather than the structure functions; since, from their model, deviations from Taylor’s hypothesis do not affect the power laws of the structure functions. For a hypothetical experiment with \( I_u \approx 0 \) we have \( \alpha F_{\alpha}(0) = \alpha \) and \( \beta F_{\theta}(0) = \beta \). However, the literature values for \( \alpha \) and \( \beta \) would not be obtained with \( I_u \approx 0 \), and Wyngaard and Clifford’s model suggests that the literature values would underestimate the “true values” for \( \alpha \) and \( \beta \) if they were determined from spectra or structure functions in conjunction with (17a) and (17b) without correction functions. Champagne et al. [1977] used Wyngaard and Clifford’s correction and obtained the true values for \( \alpha \) and \( \beta \) to be 0.50 ± 0.02 and 0.82 ± 0.04, respectively. The review by Hjort et al. [1996] suggested that the true values for these constants are \( \alpha \approx 0.52 \pm 0.01 \) and \( \beta \approx 0.80 \pm 0.03 \). An early review by Busch [1970] recommended \( \alpha \approx 0.55 \) and \( \beta \approx 0.80 \). Recently, Kaimal and Finnigan [1994] related \( \alpha \) to the von Karman constant and suggested \( \alpha \approx 0.55 \) if \( \kappa = 0.4 \). The above reviews and the mixed values for \( \alpha \) and \( \beta \) indicate that the uncertainty still persists, and to some degree, the influences of \( F_{\alpha}(I_u) \) and \( F_{\theta}(I_u) \) on temporal measured structure functions have already been included in the literature values for \( \alpha \) and \( \beta \). On the basis of this argument the necessity of applying Wyngaard and Clifford’s correction functions for determining \( \varepsilon \) and \( N_a \) using \( D_{uw}(r) \) and \( D_{ww}(r) \) is not critical.

2. In the inertial subrange the skewness factor \( S(r) \) is defined as

\[
S(r) = \left| \frac{D_{uw}(r)}{D_{uu}(r)^{1/3}} \right|^3 = \frac{4}{5} (4\alpha)^{-3/2} \quad (19)
\]

Equation (19) is useful in terms of estimating the true value of \( \alpha \) since \( S(r) \) is less sensitive to departures from Taylor’s hypothesis in highly turbulent intensity flows. Using (19), Peacock and Pond [1971] found \( \alpha \approx 0.57 \pm 0.10 \) (if the average \( S(r) \) is used, then \( \alpha \approx 0.54 \)), which is consistent with other measurements [e.g., Busch, 1973]. This value for \( \alpha \) further supports the argument in the previous paragraph.

3. Theoretically speaking, if \( I_u \approx 0 \), then \( D_{uw}(r)/D_{uu}(r) \) and \( E_{uw}(r)/E_{uu}(r) \) should be 3/4 for a locally isotropic turbulent flow (here \( E_{uw}(r) \) and \( E_{uu}(r) \) are the energy spectra for \( u \) and \( w \)). By Wyngaard and Clifford’s corrections, \( D_{uw}(r)/D_{uu}(r) \) and \( E_{uw}(r)/E_{uu}(r) \) must be systematically larger than 3/4 for \( I_u > 0 \). However, this theoretical estimate was not observed by the measurements in the literature [e.g., Hjort, 1990, Figure 1]. This implies that either Wyngaard and Clifford’s corrections are not critical, or these corrections are so small as to be within the scatter of measurements.

From the above discussion it appears that the corrections proposed by Wyngaard and Clifford may not be critical in practice. In other words, we may not obtain any improvement from applying these corrections for structure functions or spectra. We will examine this further in sections 4.1 and 4.2.
Table 1. Summary of the Experiments Above the Grass and Soil Sites

<table>
<thead>
<tr>
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<th>Experiment 1</th>
<th>Experiment 2</th>
<th>Experiment 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>July 12 to August 6, 1995</td>
<td>July 12–27, 1994</td>
<td>August 2–22, 1993</td>
</tr>
<tr>
<td>Surface cover</td>
<td>grass</td>
<td>grass</td>
<td>bare soil</td>
</tr>
<tr>
<td>Canopy height (h), m</td>
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<td>0.25</td>
<td>0.1</td>
</tr>
<tr>
<td>Zero-plane displacement ($d_0$), m</td>
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<td>~0</td>
<td>0</td>
</tr>
<tr>
<td>Roughness length ($z_0$), m</td>
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<td>0.065</td>
<td>0.002</td>
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<tr>
<td>Measurement height, m</td>
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<td>1.54</td>
<td>1.96</td>
</tr>
<tr>
<td>Sampling frequency, Hz</td>
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<td>21</td>
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<tr>
<td>Sampling period per run, min</td>
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<td>26</td>
<td>26</td>
</tr>
<tr>
<td>Number of runs</td>
<td>62 (unstable)</td>
<td>21 (stable)</td>
<td>25 (unstable), 20 (stable)</td>
</tr>
</tbody>
</table>

Notice that the roughness lengths reported here are in agreement with other reported values [Sorbjan, 1989. Table 4.1].

3. Experiment

High-frequency (21–56 Hz) wind velocities and air temperature measurements sampled above two different sites, grass and soil, were analyzed for this study. The first experiment was carried out over an Alsea Fescue grass-covered forest clearing at the Blackwood division of the Duke Forest, Durham, North Carolina. The site is a 480 m × 305 m grass site (elevation is 163 m) surrounded by a 10–12 m Lobolly pine stand. From July 12 to August 6, 1995, longitudinal, lateral, and vertical wind velocities and air temperature were sampled by a Gill triaxial sonic anemometer (path length $d$ is 0.149 m). The $x$ axis was rotated along the mean longitudinal wind direction; hence the mean lateral wind velocity was zero. The sampling frequency and duration were 56 Hz and 19.5 min, respectively, and resulted in 65,536 data points for each experimental run. The Gill triaxial sonic anemometer was set at 5.2 m above the ground and located at 250 and 200 m from the north and west end portions of the forest edge. The air temperature was determined from the speed of the sound, $C_p$, with $T = C_p^2/(A_R_0)$, where $R_0 = 287.04$ J kg$^{-1}$ K$^{-1}$ is the gas constant of dry air at constant pressure, $A_e$ = 1.4 is the ratio of the molar specific heat capacities of air at constant pressure to that at constant volume. The average grass height $h$ during this experiment was 1.07 m, and the zero-plane displacement $d_0$ ($= 2h/3$) was 0.1 m. With $u_*$ and $U$ measured, the surface roughness, $z_0$, was calculated from the mean wind profile [Brunsater, 1982, pp. 66–70]

$$U = \frac{u_*}{k} \left[ L_n \left( \frac{z}{z_0} \right) - \psi_m \left( \frac{z}{L} \right) \right] \quad (20a)$$

under near-neutral conditions ($|z/L| < 0.04$), where $\psi_m$ is defined as

$$\psi_m = \int_0^{|z_0|} [1 - \phi_m(x)] \, dx/x. \quad (20b)$$

The resulting mean $z_0$ for this experiment was 0.1 m. During July 26–27, 1994, a similar experiment was also performed. The measured mean grass height was 23 cm. The sampling frequency and duration of wind velocities and air temperature were 21 Hz and 26 min, respectively, and resulted in 32,768 data points for each experimental run. The Gill triaxial sonic anemometer was set at 1.54 m above the ground and located at 50 and 100 m from the north and west end portions of the forest edge. During this experiment the grass was sparse and short; hence $d_0$ was neglected, and $z_0$ as determined from (20a) was 6.5 cm.

The third experiment was carried out at the Campbell Track facility located at the University of California in Davis. This is a 500 m × 500 m bare soil site where $z_0 \approx 2$ mm. During August 2–22, 1993, a Gill triaxial sonic anemometer was set at 1.96 m to measure wind velocity and temperature. The sampling frequency and period were the same as the second experiment. More details regarding this experimental set can be found in the work of Katul et al. [1994]. Also, these three experiments are summarized in Table 1.

From Taylor’s hypothesis the $n$th-order structure function for flow variable ($s$) was calculated using $\langle [s(x) - s(x') \cdot 2s(x - x')] \rangle$, where $s = u, v, w$, or $\theta$. Also, each individual run was inspected to avoid any ambiguity of trends and unsteadiness in the mean meteorological conditions. With this inspection the two grass experiments resulted in 62 runs in unstable conditions and 21 runs in stable conditions; the bare soil experiment resulted in 25 runs in unstable conditions and 20 runs in stable conditions.

4. Results and Discussion

To strictly evaluate the influence of deviations from Taylor’s hypothesis in the sense of calculation of structure functions in the inertial subrange and to examine Wyngaard and Clifford’s [1977] model in practice, only the 56 Hz data (unstable runs) collected from the grass site are used since this experiment had higher sampling frequency and a wider inertial subrange when compared to the other experiments.

4.1. Identification of Inertial Subrange and Local Isotropy

The $r^{2/3}$ power law in the second-order structure functions is used to identify the inertial subrange. Figure 2 is typical for the second-order structure functions of $u$, $v$, $w$, and $\theta$ and for the third-order structure function of $u$ (for this run, $z/L = -0.22$ and $L_n = 0.33$). It is clear that the 2/3 power law exists when $d < r < z/2$ for the second-order structure functions of $u$, $v$, and $\theta$. Notice that this inertial subrange is consistent with Kaimal et al.’s [1972] measurements. A key disadvantage of using a sonic anemometer to measure structure functions is the distortion due to averaging along the finite sonic path, d. This distortion is limited to $r < d$. However, for the second-order structure function of $w$ and third-order structure function of $u$ the separation distance range for the 2/3 and 1/1 power laws is shorter. Therefore, in this study the inertial subrange is taken
as $2d < r < z/l$. It is important to note that the 2/3 and 1/1 power laws exist for all runs; this indicates that the power laws in the structure functions are not sensitive to departures from Taylor’s hypothesis.

To check the deviation from local isotropy and to evaluate Wyngaard and Clifford’s correction in the inertial subrange, the quantities $D_{ww}/D_{vv}$ and $D_{uu}/D_{vv}$ are considered (here $D_{ww}/D_{vv}$ is the averaged value of $D_{ww}(r)/D_{ww}(r)$ over the inertial subrange; $D_{uu}/D_{vv}$ is the averaged value of $D_{uu}(r)/D_{uu}(r)$ over the inertial subrange). For locally isotropic flows, $D_{ww}/D_{vv}$ should be unity (recall that Wyngaard and Clifford’s model suggests that $D_{ww}/D_{ww}$ is not influenced by the application of Taylor’s hypothesis for $I_u > 0$). Figure 3a shows the measured $D_{ww}/D_{vv}$ as a function of $I_u$ and demonstrates that $D_{ww}/D_{vv}$ is around unity and not varying with $I_u$. Theoretically, $D_{ww}/D_{vv}$ should be 3/4 if local isotropy is achieved. Figure 3b shows the measured $D_{uu}/D_{vv}$ and predictions by Wyngaard and Clifford’s model, that is,

$$D_{uu}/D_{vv} = [3F_u(I_u)]/[4F_u(I_u)] - [3 + (11/3)I_u^2]/[4 + (11/9)I_u^2],$$

as a function of $I_u$ (notice that Wyngaard and Clifford’s correction is not adequate for large $I_u$; say $I_u > 0.5$, since the assumption that the convection velocity fluctuations are Gaussian is not valid). In Figure 3b, though neither 3/4 nor Wyngaard and Clifford’s prediction agrees well with measured $D_{uu}/D_{vv}$ the scatter does not vary with $I_u$. Also, this scatter is typically found in the ASL experiments and is consistent with the results obtained by Paquin and Pond [1971]. Hence, from Figures 3a and 3b we conclude that the deviation from local isotropy is not large.


As discussed in section 2.4, Wyngaard and Clifford’s correction functions for structure functions or spectra may be unnecessary. Figures 3a and 3b further indicate that $D_{uu}/D_{ww}$ and $D_{uu}/D_{vv}$ are not sensitive to the degree of deviation from Taylor’s hypothesis in the inertial subrange for $I_u < 1.0$. Also, Wyngaard and Clifford’s prediction is not in good agreement with the measurements shown in Figure 3b. These demonstrate that in practice, Wyngaard and Clifford’s correction is not critical for structure function calculations. To further confirm this argument, (19) was used to calculate the Kolmogorov constant, $\alpha$. Figure 4 shows the scatter of 4$a$ as a function of turbulent intensity. In Figure 4 the average of 4$a$ is 2.25, which is consistent with the literature value, $4a \approx 2.2$ [e.g., Busch, 1973; Kaimal and Finnigan, 1994], and it is obvious that $a$ does not vary with $I_u$, even if $I_u \approx 1.0$ (recall that if Wyngaard and Clifford’s correction is critical, then 4$a$ determined from (19) should be larger than the literature values). Thus, on the basis of Figures 3a, 3b, and 4 and the above discussion we conclude that in the inertial subrange, calculation of structure functions is not sensitive to the degree of deviation from Taylor’s hypothesis and in practice, Wyngaard and Clifford’s [1977] corrections are not necessary. Also, from Figure 4, Busch [1973],
Figure 4. Scatter of Kolmogorov constant, $4\alpha$, as a function of turbulent intensity, $I_u$. The dashed line denotes the 2.2 value.

and Kaimal and Finnigan [1994], we decide to use $\alpha = 0.55$ ($4\alpha = 2.2$) and $\beta = 0.8$ ($4\beta = 3.2$) for this study.

4.3. Estimation of $u_*$ and $H$

As mentioned in section 2, $u_*$ and $H$ can be estimated by the traditional dissipation method (i.e., (3a) and (3b) or (5a) and (5b)) and the methods proposed by Albertson et al. [1996] and Hsieh et al. [1996], if $e$ and $N_0$ are known. Since Wyngaard and Clifford’s corrections are not necessary, $e$ and $N_0$ are determined from (16a) and (16e). Here the results for unstable and stable conditions are discussed separately.

4.3.1. Unstable conditions. Figure 5a shows the $H$ comparison between eddy correlation measurements and predictions by the traditional method, that is, (3b) in conjunction with (6b). Figures 5b and 5c are the same as Figure 5a but use (3b) in conjunction with (7b) and use (5b) in conjunction with (6b), respectively, to estimate $H$ (recall that (3b) neglects the flux divergence term in the $(\bar{\theta})/2$ budget equation but (5b) does not). We notice that $H$ predictions in Figure 5c are better than Figures 5a and 5b for moderate and strong unstable conditions. This comparison demonstrates that the flux divergence term in the $(\bar{\theta})/2$ budget equation cannot be neglected in this study. The slight underestimation in Figure 5c can be attributed to the uncertainty of the formula for $\phi_{N_0}$ and a measurements error. Figure 6 is the same as Figure 5a but uses Albertson et al.’s [1996] method (i.e., (12)) to predict $H$; the constant $A$ is 0.19, obtained from a regression analysis of (11). Using the Tsimlyansk data of 1981–1987 [Kader and Yaglom, 1990], the constant $A$ is between 0.19 and 0.25, though Albertson et al. [1996] reported a value of 0.11. Figure 6 indicates that Albertson et al.’s method predicts $H$ well for both grass and bare soil sites without suffering from the uncertainty in $\phi_0$ or $\phi_{N_0}$. Figure 7a is the same as Figure 5a but uses Hsieh et al.’s [1996] method (i.e., (13)) to estimate $H$ where $k_a$ is taken as 1.66. It is evident that Hsieh et al.’s method underestimated $H$ for large sensible heat flux. As shown in Figures 5a–5c, this is attributed to the absence of the flux divergence term in the $(\bar{\theta})/2$ budget equation in (13). To correct this underestimation,
Figure 5b. Same as Figure 5a, but uses (3b) in conjunction with (7b) for estimating $H$.

Figure 5c. Same as Figure 5a, but uses (5b) in conjunction with (8b) for estimating $H$. 
a correction function is necessary. Bradley et al. [1981] found that the ratio of production to dissipation (i.e., $\phi_{\theta}/\phi_{\theta N}$) is 1.4 for moderately unstable conditions; notice that this ratio is in good agreement with the ratio of (7b) to (8b). Hence, to correct for the flux divergence term, a correction function, $\delta(z/L)$, equal to the ratio of (7b) to (8b) is used, that is,

$$\delta(z/L) = 1.6 \frac{[3+2.5(-z/L)][1+125(-z/L)^2+78.125(-z/L)^3]}{(1+10(-z/L)+50(-z/L)^3)(10+7.5(-z/L)+6.25(-z/L)^3)}^{1/3}.$$  \hspace{1cm} (21)

In (21), what is important to note is the accuracy of the ratio and not the accuracy of the individual formulation for $\phi_{\theta}$ and $\phi_{\theta N}$. Since the formula of $\delta(z/L)$ is rather complicated, a simple and accurate correction derived from (21) is proposed in Table 2. Figure 7b presents the estimated $H$ by (13) and (21) along with (14) (here iteration is necessary for estimating $z/L$) and shows a good agreement with eddy correlation measurements. This also shows that to get a better estimation for the cases in which the flux divergence term is not negligible, (13) has to be rewritten as

$$H = \rho C_p k_u \frac{N \delta(z/L)}{\tau_0}.$$  \hspace{1cm} (22)

Figure 8a shows the $u_*$ comparison for unstable conditions between eddy-correlation measurements and predictions by the traditional dissipation method, that is, (3a) in conjunction with (6a). Figures 8b and 8c are the same as Figure 8a but use (3a) in conjunction with (7a) and use (5a) in conjunction with (8a), respectively, to estimate $u_*$ (recall that (3a) neglects the flux divergence terms when estimating $u_*$ but (5a) does not). Notice in Figures 8a–8c that measurements and predictions are in agreement, which also suggests that $\phi_{\theta N}$ and $\phi_{\theta}$ functions are valid for this study. Also, the predictions in Figures 8a–8c are similar, are consistent with Figure 1a, and suggest that it is reasonable to neglect the flux divergence terms in the TKE budget equation. Figure 9 is the same as Figure 8a but uses Hsien et al.'s method (i.e., (14) in conjunction with (22)), where $k_u$ is used as 1.1 to predict $u_*$. Notice that in Figure 9, Hsien et al.'s method predicts $u_*$ well for both grass and bare soil sites with $k_u = 1.1$, estimated by MOST with Kader and Yang's [1990] data.

4.3.2. Stable conditions. In the following figures we present the absolute value of $H$. Figure 10 shows the $H$ comparison between eddy correlation measurements and predictions by the traditional method, that is, (3b) in conjunction

<table>
<thead>
<tr>
<th>Stability Condition, $-z/L$</th>
<th>Factor, production/dissipation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–0.12</td>
<td>1.0</td>
</tr>
<tr>
<td>0.12–0.2</td>
<td>1.1</td>
</tr>
<tr>
<td>0.2–0.3</td>
<td>1.2</td>
</tr>
<tr>
<td>0.3–0.4</td>
<td>1.3</td>
</tr>
<tr>
<td>0.4–0.5</td>
<td>1.35</td>
</tr>
<tr>
<td>0.5–100</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Figure 7a. Comparison of sensible heat flux between eddy correlation measurements ($H_m$) and predictions ($H_p$) by Hsieh et al.'s [1996] method, that is, (13) in unstable conditions.

Figure 7b. Same as Figure 7a, but uses (13) and (21) along with (14) for estimating $H$. 
Figure 8a. Comparison of friction velocity between eddy correlation measurements \( \left( u_{*m} \right) \) and predictions \( \left( u_{*p} \right) \) by the traditional dissipation method, that is, (3a) in conjunction with (6a) for unstable conditions.

Figure 8b. Same as Figure 8a, but uses (3a) in conjunction with (7a) for estimating \( u_* \).

Figure 8c. Same as Figure 8a, but uses (5a) in conjunction with (8a) for estimating \( u_* \).

Figure 9. Same as Figure 8a, but uses Hsieh et al.'s \[1996\] method, that is, (14) in conjunction with (22) to estimate \( u_* \).

Good agreement between measurements and predictions is noted in Figure 12. This agreement demonstrates that neglecting the flux divergence terms in the TKE budget equation is valid for stable conditions as well. Figure 13 is the same as Figure 12 but uses Hsieh et al.'s method (i.e., (14) in conjunction with (13)) to estimate \( u_* \). Notice that Hsieh et al.'s method predicts \( u_* \) well and demonstrates that \( k_u = 1.1 \), derived from near-neutral conditions, is also suitable for stable conditions.

4.4. Comments on Stability Functions for Momentum and Heat

From MOST, \( \phi_p \), \( \phi_m \), and \( \phi_n \) are functions of \( z/L \) in unstable and stable conditions. In unstable conditions both \( \phi_p \) and \( \phi_n \) scale with \( (-z/L)^{-1/3} \). In neutral and stable conditions, field experiments show large scatter for both \( \phi_p \) and \( \phi_n \).
Interestingly, Hsieh et al.'s [1996] method leads to $\phi_u/\phi_m$ being a constant, $k_u/k_m$, for unstable, neutral, and stable conditions. This analysis suggests that although MOST may not be strictly valid for $\phi_u$ and $\phi_m$ in neutral and stable conditions, their ratio should be the same as that in unstable conditions.

The arguments for $\phi_u$ and $\phi_m$ are more complicated, since it has been observed from field measurements and generally accepted that $\phi_u$ does not follow MOST, but $\phi_m$ does (at least for moderately unstable conditions). If this is true, then (15b) is not consistent with the above accepted behavior for $\phi_u$ since (15b) suggests $\phi_u/\phi_m$ to be a constant, $k_u/k_m$. According to the directional dimensional analysis of Kader and Yaglom [1990], $\phi_u$ does follow MOST and has the same power law of $-z/L$ as $\phi_m$ in unstable conditions. Although Kader and Yaglom did not find reliable measurements of $u$ to derive an empirical formula for $\phi_u$, (15b) can be used to derive the functions for $\phi_u$ if $\phi_m$ is given. It is not our intent to argue whether $\phi_u$ follows MOST or not, what we are concerned about is that the ratio of $\phi_u$ to $\phi_m$ should be a constant, $k_u/k_m$, for any atmospheric stability condition. That is, if we accept that the measured scatter of $\phi_u$ obeys MOST, then (15b) shows from the same data set that the measured scatter of $\phi_u$ obeys MOST too. Conversely, if $\phi_m$ does not obey MOST, then $\phi_u$ does not either. This means that before any measurements are performed at a site, we are unable to conclude whether $\phi_m$ and $\phi_u$ obey MOST or not. To demonstrate this argument, measured $\phi_m$ (the measured $\phi_m$ was approximated by $k_u\varepsilon/\zeta^2 + z/L$)
was plotted as a function of z/L in Figures 14a and 14b for unstable and stable conditions, respectively. In Figures 14a and 14b the solid lines represent the eye fit estimations of \( \phi_m \) by the formulas of \((1 - 3z/L)^{1/3}\) for \(0 < z/L < 3\) and \([1 + 1/3(z/L)]^3\) for \(0 < z/L < 5\), respectively. Notice that in Figure 14a, Kader and Yaglom's [1990] 1/3 and 1/3 power laws of z/L are not observed (i.e., \( \phi_m = c_1(-z/L)^{-1/3} \) for \(0.12 < -z/L < 1.2\) and \( \phi_m = c_2(-z/L)^{1/3} \) for \(-z/L > 2.0\), where \(c_1\) and \(c_2\) are constants). By using (15b) the eye fit formulas for \( \phi_u \) should be \(2.7(1 + 3z/L)^{1/3}\) and \(2.7[1 + 1/3(z/L)]^3\) for unstable and stable conditions, respectively. Figures 15a and 15b plot measured \( \phi_m \) as a function of z/L for unstable and stable conditions, respectively. Notice that \( \phi_m \) follows the predictions made by (15b). On the basis of the discussion above we notice that for the same data set, \( \phi_m, \phi_u \) and \( \phi_u / \phi_m \) should be constant in the ASL, though the best fitted formulas (if they exist) for these individual stability functions may be different from data set to data set.

5. Conclusion

This study investigated the traditional dissipation method and the new methods proposed by Albertson et al. [1997] and Hsieh et al. [1996] with specific attention to (1) the influence of departures from Taylor's hypothesis in determining the structure functions and then the dissipation rates, (2) the influence of neglecting flux divergence terms in budget equations, and (3) the behavior of stability functions in unstable, neutral, and stable conditions. On the basis of the results and discussion we conclude the following:

1. In the inertial subrange, structure functions are not sensitive to the degree of deviation from Taylor's hypothesis, and in practice, Wyngaard and Clifford's [1977] corrections are not necessary for structure functions.

2. Our measurements show that the flux divergence terms...
can be neglected in the TKE budget equation for both unstable and stable conditions. However, although neglecting the flux divergence term in temperature variance budget equations is common in the ASI studies, our measurements demonstrate that this assumption is suitable for stable conditions but not for unstable conditions.

3. The traditional dissipation method can reproduce eddy correlation measured \( u_a \) and \( H \) if the stability correction functions are accurate for the study. With constant \( A \) equal to 0.19, Albertson et al.'s [1996] method provides good estimates for \( H \) under unstable conditions without suffering from the uncertainty in the stability correction function and iteration on \( z/L \). With the same advantages, Hsieh et al.'s [1996] method also predicts \( H \) well, under both stable and unstable conditions with constant \( k_0 = 1.66 \). However, for unstable conditions where the flux divergence term is not negligible in this study, Hsieh et al.'s method needs iteration on \( z/L \) and should be described as (22). For estimating \( u_a \), Hsieh et al.'s argument also successfully reproduces the eddy correlation measurements for both stable and unstable conditions with constant \( k_0 = 1.1.1 \).

4. Although many different formulas have been published for the stability correction functions for momentum and heat, \((\phi_v/\phi_m)k\) should be a constant, \( k_u \), and \((\phi_v/\phi_h)k\) should be a constant, \( k_u \) if the stability functions are derived from the same data set.

**Appendix: Derivation of Hsieh et al.'s [1996] method**

**Equation for Estimating \( H \)**

From (2b), \((w\theta)\), which defines the product of a characteristic vertical velocity by a temperature scale, is identical to \( N^2(\partial T/\partial z) \). On the basis of this equality and dimensional analysis [Stull, 1988, pp. 347-348] the characteristic vertical length and timescales which form a vertical velocity scale are \( z \) and \( \sigma_T^2/N \), respectively, and a proper temperature scale is \( \sigma_T \). The traditional temperature scale, \( \theta_a \), is not considered since \( \theta_a \) is not determined by the statistics of \( \theta \) only. Hence \( H = f(\sigma_T, z\sigma_T^2/N) \) and can be expressed as

\[
H = \rho C_p k_u N \frac{\sigma_T}{\sigma_a},
\]

(A1)

where \( k_u \) is a constant.

Starting with (2b) and substituting \( \phi_v \theta_a/(kz) \) for \( -\partial T/\partial z \), we get

\[
\frac{\phi_v \theta_a}{kz} = N \omega.
\]

(A2)

Substituting \( \sigma_T/\phi_v \) for \( \theta_a \) in (A2) and rearranging the terms, (A2) leads to

\[
H = \rho C_p k_u N \frac{\sigma_T^2}{\sigma_a} \sigma_T
\]

(A3)

Comparing (A3) with (A1), we have \( k(\phi_v/\phi_m) = k_u \).

**Equation for Estimating \( u_a \)**

Rearrange (2a) as

\[
u_a^2 = -\langle wu \rangle = \left( e - \frac{g}{T} \langle w\theta \rangle \frac{\partial U}{\partial z} \right),
\]

(A4)

where \( u_a^2 \) defines a product of a vertical by a horizontal velocity scale for normalizing \( u_a^2 \), a proper horizontal characteristic velocity scale is \( \sigma_u \). The traditional velocity scale, \( u_a \), is not suitable for this case, since \( u_a \) is not defined by the statistics of \( u \) only. On the basis of (A4) a proper vertical velocity scale should be formed by the vertical length scale, \( z \), and the timescale, \( \sigma_T^2/(e - \langle w\theta \rangle g/T) \). For the timescale it is not difficult to note that the denominator, \( e - \langle w\theta \rangle g/T \), is the dissipation rate of \( (u^2)/2 \) [see, e.g., Monin and Yaglom, 1971, p. 383]; hence the numerator should be \( \sigma_T^2 \) rather than \( \sigma_u^2 \). Now we can write \( u_a^2 = f(\sigma_T, z\sigma_T^2/(e - \langle w\theta \rangle g/T)) \) and express \( u_a^2 \) as

\[
u_a^2 = \frac{k_u \left( e - \frac{g}{T} \langle w\theta \rangle \right)}{\sigma_u}.
\]

(A5)

where \( k_u \) is a constant.

Starting with (2a) and substituting \( \phi_m u_a/(kz) \) for \( \partial U/\partial z \), we get

\[
\frac{\phi_m u_a}{kz} = e - \frac{g}{T} \langle w\theta \rangle.
\]

(A6)

Substituting \( \sigma_T/\phi_v \) for \( u_a \) in (A6) and rearranging the terms, (A6) leads to

\[
u_a^2 = \left( k \frac{\phi_v}{\phi_m} \right) \frac{e - \frac{g}{T} \langle w\theta \rangle}{\sigma_T}.
\]

(A7)

Comparing (A7) with (A5), we have \( k(\phi_v/\phi_m) = k_u \).

**Notation**

\( u, v, w \) fluctuations of longitudinal, lateral, and vertical wind velocity.

\( \theta, p \) fluctuations of potential air temperature and pressure.

\( U, T \) mean longitudinal wind velocity and air potential temperature.

\( \rho \) mean air density.

\( C_p \) (= 1005 J Kg \(^{-1}\) K \(^{-1}\)) the specific heat capacity of dry air at constant pressure.

\( H \) sensible heat flux, \( \rho C_p \langle w\theta \rangle \).

\( u_a \) friction velocity, \( \sqrt{-\langle uu \rangle}/2 \).

\( \theta_a \) temperature scale, \( \langle w\theta \rangle/u_a \).

\( \varepsilon, N_\theta \) mean dissipation rates of turbulent kinetic energy and 1/2 temperature variance.

\( \phi_u \) normalized function of the dissipation rate of turbulent kinetic energy, \( e/\langle uu \rangle \).

\( \phi_m \) normalized function of the dissipation rate of \( e/2 \), \( N k z (u_a, \theta_a^2) \).

\( \phi_n \) stability correction function for momentum (or normalized wind shear), \( (k_u/\sigma_u) (\partial U/\partial z) \).

\( \phi_h \) stability correction function for sensible heat (or normalized temperature gradient), \( - (kz/\theta_a) (\partial T/\partial z) \).

\( u_u \) standard deviation of longitudinal velocity, \( (u^2)^{1/2} \).

\( \sigma_u \) standard deviation of potential temperature, \( (\theta^2)^{1/2} \).

\( \phi_v \) normalized function of standard deviation of longitudinal velocity, \( \sigma_u/u_a \).
\[ \beta \text{ Kolmogrov constants for one-dimensional } u \text{ and } \sigma \text{ spectra.} \\
\kappa \text{ von Kármán constant } ( = 0.4). \\
\gamma \text{ gravitational acceleration } ( = 9.8 \text{ m/s}^2). \\
\Gamma \text{ Turbulent intensity, } \sigma_U/U. \\
d \text{ path length of sonic anemometer.} \\
R_e \text{ the gas constant of dry air at constant pressure } ( = 287.04 \text{ J kg}^{-1} \text{ K}^{-1}). \\
A_c \text{ the ratio of the molar specific heat capacities of air at constant pressure to that at constant} \\
\nu \text{ kinematic viscosity, m}^2 \text{ s}^{-1}. \\
\chi \text{ thermal molecular diffusivity, m}^2 \text{ s}^{-1}. \\
\omega \text{ twice the turbulent kinetic energy } ( = u^2 + v^2 + w^2). \\
\phi_0 \text{ normalized function of standard deviation of temperature, } \sigma_T/\theta_0. \\
z \text{ height of measurement above zero-plane displacement.} \\
d_0 \text{ zero-plane displacement.} \\
z_0 \text{ surface roughness.} \\
L \text{ Monin-Obukhov stability length, } u_L \sqrt{T'}(k g(w(0))). \]

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