An analysis of intermittency, scaling, and surface renewal in atmospheric surface layer turbulence

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Abstract

Turbulent velocity and scalar concentration time series were collected in the atmosphere above an ice sheet, a mesic grassland, and a temperate pine forest, thereby encompassing a wide range of roughness conditions encountered in nature. Intermittency and scaling properties of such series were then analyzed using Tsallis’s non-extensive thermostatistics. While theoretical links between the Tsallis’s non-extensive thermostatistics and Navier–Stokes turbulence remain questionable, the Tsallis distribution (a special interpretation of Student’s t-distribution) provides a unifying framework to investigate two inter-connected problems: similarity between scalars and velocity statistics within the inertial subrange and “contamination” of internal intermittency by “external” factors. In particular, we show that “internal” intermittency models, including the She–Leveque, Lognormal, and Log-stable, reproduce the observed Tsallis parameters well for velocities within the inertial subrange, despite the differences in surface roughness conditions, but fail to describe the fluctuations for the scalars (e.g., air temperature $\text{CO}_2$ and water vapor). Such scalars appear more intermittent than velocity when the underlying surface is a large source or sink. The dissimilarity in statistics between velocity and scalars within the inertial subrange is shown to be strongly dependent on “external” intermittency. The genesis of “external” intermittency for scalars is linked to the classical Higbie surface renewal process and scalar source strength. Surface renewal leads to a ramp-like pattern in the scalar concentration (or temperature) time series with a gradual increase (rise-phase) associated with sweeping motion from the atmosphere onto the surface or into the canopy and a sharp drop associated with an ejection phase from the surface (or the canopy) back into the atmosphere. The duration of the rise-phase is on the order of the integral time scale, while the duration of the ejection phase is much shorter and is shown to impact the distributional tails at the small scales. Implications for “scalar turbulence” models are also discussed in the context of biosphere–atmosphere $\text{CO}_2$ exchange.

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1. Introduction

The scalar exchange between the land surface and the atmospheric boundary layer (ABL) remains an urgent yet notoriously difficult problem to solve. Almost half a century of research on scalar turbulent flows within the ABL focused on two primary thrusts: one that deals with the integral properties of scalar transport with an emphasis on how fluxes and concentration vary in space and time; the other deals with inertial and dissipation scales and seeks to investigate possible universal scaling behavior. While the former is impacted by boundary conditions imposed on the flow domain, the latter assumes that small-scale eddies achieve a statistically independent state from large scales at very high Reynolds number (characterizing atmospheric flows). Experimental evidence on scalar turbulence to date [1,2] shows that large and small scales are strongly coupled, so treating them independently may not be a desirable approach for progress on scalar exchange between the land surface and the atmosphere. Moreover, there is growing theoretical and experimental evidence that passive scalar concentration
fluctuations can exhibit complex dynamical behavior far richer than its turbulent velocity counterpart at high Reynolds number [1–7]. In fact, several studies have documented that, at high Reynolds number, scalar statistics rarely achieve universal scaling at fine scales even if the velocity appears to exhibit universal scaling [1,2,4].

While fully exploring these theoretical aspects is well beyond the scope of a single study, we focus on whether Tsallis distributional parameters can inform us about two inter-related problems: similarities between velocity and scalar statistics at a wide range of scales, and the interaction between “external” and “internal” intermittency buildup at finer scales. These two issues are now receiving broad attention in physics and atmospheric sciences, though the use of Tsallis-type statistics to explore them jointly has received less attention [8,9]. Theoretical links between Navier–Stokes turbulence and non-extensive entropy measures remains questionable [10]; however, the Tsallis distributions do offer a flexible “diagnostic” tool to jointly explore questions related to intermittency and scaling of high-Reynolds-number flows in the atmosphere above natural surfaces.

After Tsallis [11] introduced an entropy based measure \( S_q \) that describes non-extensive statistical equilibrium, numerous attempts have been proposed to use this measure to turbulent flows have been [8–10,12–20]. In its basic form, such a measure is

\[
S_q = \frac{1}{1-q} \left( 1 - \int_{-\infty}^{+\infty} (p(x))^q \, dx \right) \quad (1)
\]

and its maximum value is achieved when the probability density function \( p(x) \) is given by

\[
p(x) = \frac{1}{Z_q} \left( 1 - \beta(1-q) x^2 \right)^{\frac{1}{1-q}} \quad (2)
\]

where \( x \) is the energy state (normalized to have a unit variance), \( q \) is the non-extensive entropic parameter, \( Z_q \) is a normalizing constant, and \( \beta \) is the Lagrange parameter related to \( q \) (for \( q \in [1,5/3] \)) by

\[
\beta = \frac{1}{5-3q} \quad (3)
\]

when normalizing \( \bar{x}^2 = 1 \), where the overbar is the time-averaging operator [9,12]. The non-extensive \( q \) parameter in Eq. (2) is often used as a measure of information incompleteness. For \( q \geq 3 \), \( p(x) \) does not exist; for \( 3 > q \geq 2 \) it is a Lévy-stable distribution, and for \( q \to 1 \), \( p(x) \) becomes Gaussian. Finally, when \( q < 1 \), the distribution resembles a cutoff process. For turbulent fluctuations in high-Reynolds-number flows, \( q \in [1,5/3] \), as evidenced by several experimental studies [9,12].

We address the two study objectives by first establishing links between the Tsallis \( q \) and standard “internal” intermittency models of turbulent velocity. The terms “internal” and “external” are used to distinguish between intermittency buildup originating from the mean turbulent kinetic energy dissipation rate and exhibiting universal scaling within the inertial subrange, and intermittency resulting from interaction between the flow and externally imposed boundary conditions and not exhibiting universal scaling for all turbulent flows.

We use time series measurements of turbulent velocity components and several scalars collected over a wide range of surface roughness conditions to determine similarities in \( q \) among flow variables and the dependence of \( q \) on boundary conditions. Comparing how well the internal intermittency models predict \( q \) across a wide range of scales and surface roughness conditions permits us to assess whether boundary conditions known to impact large-scale eddies interact with fine-scale turbulence. We then proceed to compare the \( q_s \) for velocity and several scalars within the inertial subrange. Departures between the scalar and velocity \( q_s \) are further analyzed within the context of boundary conditions for scalars, with particular attention to organized motion whose genesis is the surface renewal process. The applicability of Tsallis distributions to scalar transfer in atmospheric flows is also discussed.

2. Theory

In turbulence, it is customary to evaluate scaling and intermittency buildup using velocity increments [21]. Let \( x = \Delta_r u, \Delta_r w, \Delta_r c \) where \( u \) and \( w \) are the longitudinal and vertical turbulent velocities and \( c \) may be air temperature \( T \), water vapor concentration \( H_2O \) or carbon dioxide \( CO_2 \), and \( \Delta_r \) is the differencing operator for an arbitrary turbulent flow variable \( \psi \) such that \( \Delta_r \psi = \psi(\hat{x} + r) - \psi(\hat{x}) \), where \( \hat{x} \) is an arbitrary position in the flow and \( r \) is the separation distance between two points, often used as a scale surrogate. It can be shown [12,13] for the Tsallis distribution in Eq. (2) that the kurtosis

\[
K(r) = \frac{\bar{x}^4}{(\bar{x}^2)^2} \quad (4)
\]

is related to the Tsallis parameter \( q \) as

\[
q(r) = \frac{15 - 7K(r)}{9 - 5K(r)} \quad (5)
\]

Hence, at each \( r \), \( K \) can be measured from time series and \( q(r) \) can be determined. Eq. (5) suggests that, for very large \( K \) (i.e. \( K \gg 3 \)), \( q(r) \to \frac{7}{2} \), independent of \( r \). Within the inertial subrange, a range much smaller than the integral length scale but much larger than the Kolmogorov viscous dissipation length scale, \( K(r) \) can be estimated from standard “internal” intermittency models for \( \Delta_r u^n = C_n r^{\delta_n} \), where \( \delta_n \) are the scaling exponents. The internal intermittency buildup was extensively studied and various models, reviewed in Table 1, have been proposed to quantify its effect on inertial subrange scaling. All the models in Table 1 neglect the flow boundary conditions and assume that fine-scale turbulence attains a universal state that is independent from the production mechanism.
Table 1
Models for the “internal” intermittency exponent $\xi_n$ for $\Delta r u^n = C_n r^{\xi_n}$ within the inertial subrange, where $n$ is the order of the moment

<table>
<thead>
<tr>
<th>Model</th>
<th>$\xi_n$</th>
<th>Parameter range</th>
</tr>
</thead>
<tbody>
<tr>
<td>K41 [52];</td>
<td>$\xi_n = \frac{n}{3}$</td>
<td>No intermittency</td>
</tr>
<tr>
<td>K62 or Log-normal [53]</td>
<td>$\xi_n = \frac{n}{3} + \frac{n^3}{18} \left(3 - n^2\right)$</td>
<td>$\mu = 0.10-0.25$ [27]</td>
</tr>
<tr>
<td>Mono-fractal model [21]</td>
<td>$\xi_n = \frac{n}{3} + (3 - D) \left(1 - \frac{n}{3}\right)$</td>
<td>$D = 2.78$</td>
</tr>
<tr>
<td>Bi-fractal model [21]</td>
<td>$\xi_n = \begin{cases} \frac{n}{3}; n \leq 3 \ \frac{n}{3} + (3 - D) \left(1 - \frac{n}{3}\right); n &gt; 3 \end{cases}$</td>
<td>None</td>
</tr>
<tr>
<td>She–Leveque [55]</td>
<td>$\xi_n = \frac{n}{3} + 2 - 2 \left(\frac{2}{7}\right)^{n/3}$</td>
<td>$\lambda = 1.33$</td>
</tr>
<tr>
<td>Log-stable [27]</td>
<td>$\xi_n = \frac{n}{3} + 1 - \log_2 \left(\lambda^{n/3} + (2 - \lambda)^{n/3}\right)$</td>
<td></td>
</tr>
</tbody>
</table>

With $\Delta r u^n = C_n r^{\xi_n}$, $K(r)$ within the inertial subrange can be computed from [14]:

$$K(r) = \frac{\Delta r u^n}{(\Delta r u_r^2)^{1/2}} = C_n \frac{r^{\xi_n}}{C_L^2 r_{2L_2}^2} = K_L \left(\frac{r}{L}\right)^{\alpha}$$  \hspace{1cm} (6)

where $\alpha = \xi_4 - 2 \xi_2$, and $K_L$ is the kurtosis at a reference scale $r = L$ where $L \leq z_m$, and $z_m$ is the measurement height from the boundary. Notice that Eqs. (6) and (5) can be used to link $q$ to $\xi_4 - 2 \xi_2$ explicitly. This linkage implies that $q(r)$ can be thought of as a “local” scaling parameter linking “internal” intermittency buildup at small scales with the tails of $p(x)$.

To illustrate, consider the case of the Log normal (K62) model, for which $\xi_n$ is characterized by the internal intermittency parameter $\mu$ (Table 1) as

$$\xi_n = \frac{n}{3} + \frac{\mu}{18} \left(3n - n^2\right).$$

The parameter $\mu$ can readily be determined from $q(r)$ by regressing $Y = \log \left(\frac{9q(r) - 15}{5q(r) - 7}\right)$ upon $X = \log(r)$, using the regression model

$$Y = B - \frac{4}{9} \mu X$$  \hspace{1cm} (7)

Hence, the Tsallis $q$ can be used to link the distribution in Eq. (1) with the standard intermittency model parameters reviewed in Table 1.

3. Data

To address the two study objectives, we use three published atmospheric turbulence data sets collected over a wide range of surface roughness values under mostly near-neutral stability conditions. The first dataset was collected in Antarctica above an ice sheet [22–25] (i.e. almost a smooth surface); the second one was collected above 0.5 m tall mesic grassland [26,27]; the third one was collected above a 17 m tall managed, temperate Loblolly pine plantation [28–32]. We chose this wide range of surface roughness values because it is conceivable that surface boundary conditions can impact large-scale eddy motion, and large-scale eddy motion can directly interact with fine scales without resorting to the standard energy cascade. The impact of such interaction is often realized as an increase in intermittency parameters of the $\xi_n$ models (e.g. $\mu$ values significantly in excess of 0.25). Such a data set should permit us to detect whether boundary conditions do affect the statistics of fine-scale turbulence within the inertial subrange.

The three velocity components were collected using standard sonic anemometry at all sites; the air temperature was collected above the grass surface and the pine forest using the sonic anemometer speed of sound, and $H_2O$ and CO$_2$ turbulent concentration fluctuations were only measured at the pine forest using an open path infrared gas analyzer (LI-7500, Licor, Lincoln, Nebraska). All three data sets exhibit an approximate inertial subrange identified as the narrower of two conditions: (1) linearity with $r$ of the third-order structure function, and (2) approaching 4/3 of the ratio of the second-order structure functions for $w$ and $u$. Using these two criteria, we found that a conservative limit for the inertial subrange was bounded by $2 d_{sl} < r < \frac{1}{2} \left(z_m - \frac{2}{7} h_c\right)$, where $d_{sl}$ is the sonic anemometer path length ($=10–15$ cm, depending on the experiment), and $h_c$ is the canopy height (Table 2). Because Taylor’s hypothesis was used to convert time $t$ into $r$, only runs with a squared turbulent intensity $I_u^2 = \left(\frac{u^{1/3}}{U}\right)^2 < 0.1$ were used, where $U$ is the mean longitudinal velocity and $\sigma_u$ is the standard deviation of $u$. This limit minimizes any flow distortions in scale due to the application of Taylor’s frozen turbulence hypothesis [33].

4. Results and discussion

4.1. Tsallis parameters and measured data

Before proceeding to the two study objectives, we assess how well Eq. (2) describes the velocity and scalar time series statistics at the three sites. This comparison is conducted as follows: using measured $x^3$ at each $r$ and for each flow variable, we determine $K(r)$ from Eq. (4) and $q(r)$ from Eq. (5), and then we proceeded with the comparison between modeled $p(x)$ from Eq. (2) with measured $p(x)$ at each $r$. From these calculations, we address the first objective by contrasting the measured $q(r)$ and $K(r)$ for each flow variable ($u$, $w$, and $c$). The second
Table 2
Summary of the experimental setup at the three sites

<table>
<thead>
<tr>
<th>Variable/surface</th>
<th>Ice surface</th>
<th>Grass</th>
<th>Loblolly pine forest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site location</td>
<td>Antarctica</td>
<td>Duke Forest, near Durham, NC, USA</td>
<td></td>
</tr>
<tr>
<td>Vegetation height $h_c$ (m)</td>
<td>0</td>
<td>0.5</td>
<td>17</td>
</tr>
<tr>
<td>Measurement height $z_m$ (m)</td>
<td>10</td>
<td>5.1</td>
<td>20.2</td>
</tr>
<tr>
<td>Sampling frequency $f_s$ (Hz)</td>
<td>20.8</td>
<td>56</td>
<td>10</td>
</tr>
<tr>
<td>Sampling period per run (min)</td>
<td>26.5</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Flow variables measured</td>
<td>$u, w$</td>
<td>$u, v, w, T$</td>
<td>$u, v, w, T &amp; CO_2 &amp; H_2O$</td>
</tr>
</tbody>
</table>

Fig. 1. Comparison between measured (open circle) and modeled (dots) $p(x)$ for separation distances $r/z_m = 0.059, 0.12, 0.2, 0.4, 0.98, 1.96, 4.9$ and for the longitudinal ($u$) (left) and vertical ($w$) (middle) velocities, and air temperature ($T$) differences at the grass site. Note that for each flow variable, $p(x)$ is shifted upwards by four decades with increasing $r/z_m$, with the lowest corresponding to $r/z_m = 0.059$. Also, for each $r/z_m$, the 12 separate lines and data correspond to 12 runs collected under similar meteorological and mean wind conditions, each 1/2 h in duration.

Objective is addressed by comparing predictions from Eq. (6) using all the $\xi_n$ models with the measured $K(r)$ and $q(r)$. If $K(r)$ and $q(r)$ exceed predictions from Eq. (6), then “external” intermittency has a significant impact on the energy cascade.

Fig. 1 compares measured and modeled $p(x)$ for $u, w$, and $c = T$ at the grass site. Similar agreements were found for the other two sites. From Fig. 1, it is clear that the Tsallis modeled $p(x)$ reproduces the measured $p(x)$ well for a wide range of $r/z_m$ (i.e. inertial subrange scales and larger) in the case of the two velocity components. For the temperature, a model bias is evident — an underestimation of the tails at small $r/z_m$ and for negative $x$. Fig. 1 also shows that the measured $p(x)$ is heavy-tailed at small scales (or $r$) and approaches Gaussian for larger scales for all three variables. This comparison suggests that the Tsallis $p(x)$ offers a unifying framework to analyze both scalar statistics (intermittency and similarities in velocity) using a single parameter ($q$).

Figs. 2a and 2b compare the measured and modeled $K$ and $q$ for the Antarctica and grass sites, respectively, using the models in Table 1. The measured $q$ varies from a maximum of 1.3 (at the smallest scale) and approaches unity (i.e. Gaussian) with increasing $r$. Recall that, for large $K(r)$, $q \rightarrow 1.4$ and becomes independent of $r$.

We compare $q$ from these experiments to recent high-resolution Lagrangian experiments conducted on small-scale statistics of the acceleration ($a$) of a particle in high-Reynolds-number turbulence. These experiments were shown to be well described by Tsallis statistics with a $q = 3/2$ [12]. In high-Reynolds-number (Eulerian) flow with small $I_u \ll 0.1$, the Lagrangian $a$ can be linked to a Eulerian $\Delta_u$ by

$$a = \frac{\text{d}a(t)}{\text{d}r} \approx \frac{u(t + \tau) - u(t)}{\tau} \approx \frac{\Delta_u}{r} \left( rU \right)$$

where $\tau$ is a time scale. At a specified $r$, $q$ obtained from $\Delta_u$ also reflects its Lagrangian acceleration value. In the Lagrangian experiments, $r$ (or $\tau$) were based on viscous dissipation length and time scales [12], thereby providing an upper limit on the Eulerian $q$ within the inertial subrange.
Fig. 2a. Comparison between measured and modeled Tsallis statistics for the Antarctica velocity data using the models in Table 1 for ξ_n. The solid horizontal line is for K41.

Fig. 2b. The same as Fig. 2a, but for the grass site.

our experiments, r > d_m = 10 cm, which is much larger than the Kolmogorov microscale (~0.1 mm). All the data from the experiments here exhibit q < 1.5 and are consistent with the upper limit set by the Lagrangian experiments.

The measured K is clearly not constant within the inertial subrange (as predicted by K41) and appears highly non-Gaussian at small r, consistent with Fig. 1. It is also clear that K62 with μ = 0.25, the She–Leveque (no parameters) and the Log-stable (λ = 1.33) models reproduced the measured K and q well for the u time series. We used the regression model in Eq. (7) to infer μ from the u time series and found μ ≈ 0.24, which is strikingly similar to the μ = 0.227 obtained from hot-wire measurements in a round free jet (obtained by evaluating ξ_n up to n = 7) and μ = 0.20–0.25 obtained from other atmospheric measurements [1,34].

The w series appeared more intermittent than u at both the ice and the grass surfaces — though it remained bounded by K62 and the mono-fractal model. Furthermore, the measured q and K for temperature were consistently larger than their velocity counterparts, a clear indication of some dissimilarity between scalar and velocity statistics within the inertial subrange. This dissimilarity in q between temperature and velocity was also reported above an Amazonian rain forest [9] for unstable atmospheric stratification but not for mildly stable stratification (we revisit this point later regarding possible reasons). Moreover, for the grass site, the temperature K, q were larger than what any of the “internal” intermittency models predicted. Given the linkage between μ and q, this finding is not entirely surprising as a few studies have already reported a μ (for the K62 model) from temperature data on the order of 0.35–0.40, while μ from the concomitant u measurements was about 0.25 [35].

4.2. Similarity in scalar-velocity statistics

From Fig. 2b, it is tempting to argue that temperature, being an active scalar (i.e. a scalar that affects turbulent velocity via buoyancy production in the turbulent kinetic energy budget), may impact the small-scale intermittency [36,37]. We show next that this effect is not the main reason here. To explore this point, we use data sets from the pine forest, because it includes multiple scalars: some active (e.g. temperature) and some passive (e.g. CO₂). To illustrate this, we show one 30 min time series run with all the velocity and scalar flow variables in Fig. 3. This run was collected between 18.00 and 18.30 when photosynthetically active radiation (PAR) was low; leaf photosynthesis was almost absent (i.e. no sink of CO₂ by the canopy), and leaf stomata were practically closed (i.e. negligible H₂O source from plant transpiration). The measured turbulent fluxes of heat and water vapor above the canopy were <5% of their maximum daytime value. On the other hand, the CO₂ flux was large in magnitude. The reason why the CO₂ flux is large (and positive), despite stomatal closure, is because the forest floor and the above-ground biomass continuously respires, thereby enriching the within-canopy air space with CO₂.

Visual inspection of Fig. 3 clearly shows that the normalized time series of the CO₂ concentration above the canopy is dominated by ejections of CO₂ enriched air that appear to produce an “on–off” process (or large external intermittency in the time series). This “external” intermittency is visually absent in the velocity, and less evident in the temperature and water vapor time series of Fig. 3.
Fig. 3. Time series of the three velocity components, air temperature, CO$_2$, and water vapor above a pine forest for near-neutral conditions. All variables are normalized to have zero mean and unit variance. The run is collected between 1800 and 1830 when photosynthetically active radiation (PAR) $\sim 0$. 

Fig. 4 shows the Tsallis statistics for this run and reveals significant differences in $q$ between CO$_2$ and the remaining flow variables. It is clear that “internal intermittency” models listed in Table 1 capture the $r$ dependence of $K$, $q$ well, within the inertial subrange for the velocity components. The $K$ and $q$ estimates for temperature and water vapor are indistinguishable in the inertial subrange and appear more intermittent than their velocity counterparts. However, the $K$ and $q$ for CO$_2$ suggest a much more intermittent process than temperature or water vapor. When the results in Figs. 3 and 4 are taken together, it is clear that external intermittency, primarily driven by the CO$_2$ source strength within the canopy, did impact the “internal” intermittency of CO$_2$ within the inertial subrange (defined by the velocity). A logical question then is whether land surface scalar fluxes, external and internal intermittency, and similarity in scalar-velocity statistics can be linked.

4.3. Higbie’s surface renewal: A link between external and internal intermittency

As early as 1935, Higbie [38] proposed a surface renewal theory [39] to investigate interfacial heat transfer between a liquid and a gas; this approach has now gained some popularity in micrometeorology and surface hydrology [40–46]. Higbie visualized heat transport as occurring by the arrival of fresh transporting fluid elements from the bulk fluid above a heated surface, followed by unsteady diffusion transport during contact (or residence time) between these fluid elements and the surface, and finally the eventual replacement of this stale element by a fresh fluid from aloft.

Within canopies, Fig. 5 shows schematically how the surface renewal leads to a “ramp-like” motion (the source of external intermittency) for CO$_2$, at least from a Lagrangian perspective. The inset in Fig. 5 is 200 s of CO$_2$ concentration time series taken from Fig. 3 to “zoom-in” on the ramp-like structural features. For the grass site, air temperature also exhibits ramp-like patterns (not shown here) during daytime when the sensible heat flux is positive. The existence of sharp edges from ramps and their potential impact on the structure functions have already been documented in field and laboratory studies [1, 47–49]. To cite Warhaft’s review [1]: “While the ramp-cliff structures are large-scale features, on the order of an integral scale, the front itself is sharp, and thus is manifested at the small scales”.

Numerous experiments reviewed in Warhaft [1] suggest that the sharp front becomes even sharper at higher Taylor microscale Reynolds numbers, thereby compounding its effects on inertial subrange statistics as one shifts from laboratory to atmospheric surface layer experiments. How these sharp edges affect intermittency and the Tsallis parameters are explored next using a simple phenomenological example.

Consider a fractional Brownian motion (fBm) time series ($y_{fBm}$) with a Hurst exponent $H = 1/3$ (to model K41
turbulence with no internal intermittency) and a ramp-like periodic signal \( y_{\text{RAMP}} \) with a ramp length (or residence time) of about 60 s. Both time series are normalized to have zero mean and unit variance (Fig. 6a). Next, define a composite signal by

\[
y(t) = \alpha y_{\text{fBm}} + (1 - \alpha) y_{\text{RAMP}},
\]

where \( \alpha \in [0, 1] \) is a parameter to weigh the relative importance between underlying turbulence and ramp-like structures. Time series of \( y(t) \) are displayed for a few \( \alpha \) in Fig. 6b. For \( \alpha = 0.1 \), the composite signal is primarily dominated by ramps and the computed \( K(r) \) approaches 200. In Fig. 6b, we present the computed \( K(r) \), not \( q(r) \), because, for large \( K \) (e.g. 200),
Fig. 6a. A ramp-like series with a constant residence time of 60 s and a fractional Brownian motion (fBm) series with a Hurst exponent $= 1/3$. The fBm series recovers K41 scaling (no internal or external intermittency). $q(r)$ saturates at 7/5, as mentioned earlier. Also, when $y(t)$ is dominated by ramps, $p(x)$ is far from being symmetrical, as predicted by Eq. (1). For $\alpha = 1$, the composite signal reduces to a standard fBm and K41 statistics are recovered as expected (i.e. $K = 3$ and $q = 1$ both independent of $r$). As $\alpha$ increases from 0 to 1, $K(r)$ shows that internal intermittency within the inertial subrange (defined as $r/z_m < 0.5$) becomes less and less coupled with external intermittency (i.e. ramp-like motion). Interestingly, for $\alpha = 0.65$, $K(r)$ of the composite time series closely matches the measured $K(r)$ for CO$_2$.

Notice that, in Figs. 5 and 6, the duration of the ramp (or residence time) is on the order of 60 s and, for $\bar{U} \sim 1$ m s$^{-1}$, leads to a mean eddy size of 60 m that is much larger than $z_m$. However, the ejection-phase and subsequent renewal process occurs in about 0.4 s (Fig. 5), corresponding to a spatial scale of about 0.4 m, well within the inertial subrange and much smaller than $z_m$. In the Tsallis $p(x)$ framework, such short-lived ejections contribute to large $x$ (though negative) that are larger than that predicted by Eq. (1) at small $r$ (Fig. 1).

It was pointed out that finite structure skewness $S = \frac{\bar{x}^3}{(\bar{x}^2)^{3/2}}$ within the inertial subrange can be interpreted as a signature of contamination from larger scales [1,7] because small-scale isotropy necessitates $S(r) = 0$. Within the Tsallis framework, the zero-skewness constraint imposed by local isotropy is automatically satisfied by virtue of distributional symmetry. In Fig. 7, we show the comparison between measured and modeled $p(x)$ for CO$_2$ and $u$. It is clear that the Tsallis modeled $p(x)$ under-estimates the negative excursions of $x$ for CO$_2$ (at least when compared to $u$) consistent with the ramp models (and the finite skewness). The $p(x)$ comparisons in Fig. 7 for CO$_2$ are also consistent with the results for temperature in Fig. 1, further confirming the dominant role of surface renewal on scalar-velocity dissimilarity and internal intermittency buildup.

Finally, we note that the surface renewal hypothesis can explain the recent findings above an Amazonian rain forest [9] that reported a larger $q$ for temperature than vertical velocity under unstable conditions (i.e. the surface is a source of heat and ramp-like patterns existing in the temperature). For mildly

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Fig. 6b. Left: Composite time series constructed from linear combinations of ramp-like ($y_{RAMP}$) and fBm ($y_{fBm}$) signals using $y = (1-\alpha)y_{RAMP} + (\alpha)y_{fBm}$. Right: The computed $K(r)$. The measured $K(r)$ for CO$_2$ is taken from Fig. 4 (circles) and is repeated for reference. A decreasing $\alpha$ corresponds to an increase in “external” intermittency produced by the surface renewal process.
stable conditions (i.e., the surface is cooling gradually and a weak inverse ramp-like motion may exist), these experiments report \( q \) for temperature comparable to its vertical velocity counterpart.

5. Conclusions

Using velocity and scalar measurements in the atmosphere above ice, grass, and forest surfaces, we showed that the Tsallis distribution reproduces the measured velocity differences well, thereby offering a unifying framework for analyzing similarities and differences in scalars and velocity, and external versus internal intermittency. Internal intermittency models, including the She–Leveque, K62 with \( \mu = 0.25 \), and the Log-stable model, reproduce the Tsallis parameters for velocity well but not for scalars within the inertial subrange. It was shown that scalars are more intermittent than velocity, at least when the boundary is a large scalar source or sink. This dissimilarity between velocity and scalar statistics in terms of internal intermittency parameters was shown to be strongly dependent on external intermittency. The genesis of the “external” intermittency in scalars is the ramp-like motion linked to both the surface renewal process and the scalar source strength. This ramp-like motion is absent in the velocity time series. Numerical experiments demonstrate that a strong ramp-like signature can “contaminate” inertial subrange scaling, because the “ejection” phase is very short-lived and contributes significantly to the tails of the scalar concentration increments at small separation distances. Stated differently, ramp-like patterns in the scalar concentration time series are the result of interactions between the flow and its boundary conditions (exogenous) yet their impact can be realized as excursions from the universal scaling properties of small-scale turbulence (Table 1). Hence, ‘scalar turbulence’ may prove to be a logical ‘bench-mark’ phenomenon for exploring connections between exogenous and endogenous interactions, now the subject of numerous investigations in systems with long-range persistence and memory [50,51].

The broader impact of this work shows under which conditions passive scalar concentration fluctuations exhibit complex dynamical behavior richer than its turbulent velocity counterpart, and why scalar statistics rarely achieve universal scaling at fine scales, even when the velocity appears to exhibit universal scaling.

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